An Investigation into Bray's Heuristics for Mathematical Learning Activities as Applied to Functions

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Declaration

I declare that the work described in this Dissertation is, except where otherwise stated, entirely my own work and has not been submitted as an exercise for a degree at this or any other university.

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Abstract

Mathematics is an important activity for individuals and society (Hoyles, 2016, p. 225) with knowledge of mathematics and qualifications in it becoming ever more important gateways to life-skills, higher levels of education and to the success of economies (Norris, 2012, p. 22).

International evaluations of mathematics achievement have found that after finishing basic education many students' knowledge and competencies are below the expected level. There are many who see the subject as boring and irrelevant (Hoyles, 2016, p. 225) and these negative attitudes are a concern as it has been found that attitudes to the subject have a large bearing on the level of achievement of students (Boaler, 1993; Kislenko, Grevholm, & Lepik, 2005; Ma, 1999). This research focusses on the attitudes of student engagement with, and confidence in, mathematics.

Mathematics is vast in terms of choices of topics to be included in school curricula. Functions are included in all curricula internationally (Hodgen, Pepper, Sturman, & Ruddock, 2010; NCCA, 2013b, 2013c; NCTM, 2000) as they are seen as useful for consolidating students algebraic understanding, essential in preparing students for the topic of calculus (Lagrange, 2014) and important for helping students make sense of the world around them (Kalchman & Koedinger, 2005).

For this research, Bray's set of design heuristics (Bray, 2015) have been chosen as the basis for designing a rich learning experience in the topic of functions as these heuristics have been found to increase student engagement with, and confidence in, mathematics. Bray's interventions were conducted outside of the normal school timetable. This study builds upon Bray's work by using a web-based Toolkit[1] with social constructivism at the core of its design, to create a learning experience with a high level of technological integration within the confines of a normal school timetable. The research focus is how participation in a rich learning experience, designed in line with Bray's heuristics, improves student engagement and confidence when applied to the topic of functions within the confines of a normal school timetable.

The research methodology used was an exploratory case study. 24 students engaged in the learning experience over 8 hours and 40 minutes. Pre- and post- learning experience data on engagement and confidence was collected through the use of the Mathematics and Technology Attitudes Scale (MTAS) (Pierce, Stacey, & Barkatsas, 2007). Data was also collected via researcher observation, through analysing the data captured by the technology and a number of group interviews. The study found that the learning experience had a positive impact on the attitudes of participants, especially in their feelings about mathematics (affective engagement) and their attitude to learning mathematics with technology.

[1] A Toolkit describes technologies that are designed in accordance with a specific pedagogical approach, that provide support for the student and the teacher through tasks and lesson plans, and provide feedback for assessment (Bray & Tangney, under review, p. 10).

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1 Introduction

1.1 Motivation for the Research

Mathematics is an important activity for individuals and society (Hoyles, 2016, p. 225) with Norris (2012, p. 22) saying knowledge of mathematics and qualifications in it are becoming ever more important gateways to life-skills, higher levels of education and to the success of economies.

International evaluations of mathematics achievement, such as TIMMS and PISA, have found that many students' knowledge and competencies are below the expected level when they finish basic education. These evaluations have also found that even students with satisfactory results in mathematics do not like the subject and do not see why so much time should be spent studying it (UNESCO, 2012, p. 9). This is a major concern as it has been found that attitudes to the subject have a large bearing on the level of achievement of students (Boaler, 1993; Kislenko et al., 2005; Ma, 1999). There are many who see the subject as boring and irrelevant (Hoyles, 2016, p. 225). Researchers suggest that students who see the subject as boring will not engage with the subject nor increase their understanding of the subject (Boaler, 1993, p. 346; Kislenko et al., 2005). As attitudes to the subject are important factors that can affect students' achievement levels in mathematics they have been chosen as the focus of this study. In particular, this research focusses on improving the attitudes of student engagement with, and confidence in, mathematics.

Mathematics is vast in terms of choices of topics to be included in school curricula. Functions are included in all curricula internationally (Hodgen et al., 2010; NCCA, 2013b, 2013c; NCTM, 2000) as they are seen as useful for consolidating students algebraic understanding and also essential in preparing students for the topic of calculus (Lagrange, 2014). Functions are also seen as important for helping students make sense of the world around them (Kalchman & Koedinger, 2005). They are essential to undergraduate mathematics, modern mathematics, and a deep understanding of functions is also essential for future scientists, engineers and mathematicians (Yerushalmy & Shternberg, 2001).

Approaches to teaching functions are discussed in Chapter 2. These are quite varied and range from using technology or hands-on materials to associating functions with kinaesthetic movement (Anabousy, Daher, Baya'a, & Abu-Naja, 2014). This study is anchored in the area of technology because technology can be used, in conjunction with other factors, to improve a variety of learning experiences. There are many choices in terms of how one can use technology in schools. In particular, this research uses the work of Bray and Tangney whose analysis of recent studies of technological interventions found that combining a "constructivist, team-based, project-based pedagogic approach, and non-standardised assessment methods (Ertmer & Ottenbreit-Leftwich, 2010; Li & Ma, 2010; Voogt & Pelgrum)" produces the greatest positive effects (Bray & Tangney, under review, p. 3).

In particular, Bray's set of design heuristics (Bray, 2015, p. 67), which are "desirable attributes of technology-mediated mathematics learning activities that have the potential to increase student engagement and confidence", have been chosen as the basis for designing a rich learning experience in the topic of functions.

Bray developed the heuristics through applying them to a variety of topics including statistics and probability, geometry and trigonometry, number, and functions (Bray, 2015, p. 9). Bray had two phases to her research; an exploratory phase and an explanatory phase. During the first phase the interventions took place outside of school in the Bridge21 Learning Laboratory[2]. The students involved were all familiar with the Bridge21 lesson activity structure – a structure that encourages collaboration and peer learning - and had all volunteered to take part in the interventions. During the second phase the four interventions took place in school settings. The students involved were all familiar with the Bridge21 lesson activity structure and had not volunteered to take part in the interventions. The in-school interventions were for (i) two hours a day for a week, (ii) two days from 10am to 4pm each day, (iii) two hours in a single afternoon and (iv) two hours a day for a week. A normal school timetable commonly consists of 40 minute or 80 minute periods.

The motivation for this research comes from one of Bray's own recommendations for future research. Bray recommended more in-school interventions to see whether her findings could be "replicated - both for repeated use with similar students, and for greater numbers of classes following syllabi leading to state examinations" (2015, p. 172). This study is an in-school intervention of a comparable duration to Bray's in-school interventions. The learning experience uses activities that are curriculum aligned and conducted using class periods of either 40 minutes or 80 minutes in length. This study significantly builds upon Bray's work as the heuristics were given a robust test within a normal classroom setting. The research question is can participation in a rich learning experience, designed in line with Bray's heuristics, improve student engagement and confidence when applied to the topic of functions within the confines of a normal school timetable?

1.2 Design

The learning experience which is the focus of this study applies each of Bray's five design heuristics to create a learning experience with a high level of technological integration. The type of technology used was a web-based *Toolkit* with social constructivism at the core of its design. Having social constructivism in the design of the technology itself complemented two of Bray's heuristics. The first of these is that the learning should be team-based, collaborative and use a socially constructivist

[2] The Bridge21 Learning Laboratory is a specially designed learning space "for group learning with breakout areas and alcoves, each facilitating information exchange, team collaboration and individual reflection".

approach and the second is that a model of 21st century learning should be used such as the one used in this learning experience; the Bridge21 model.

The activities for the learning experience included important ideas for functions, including multiple representations, modelling situations with functions and analysing the effects of changing parameters when learning about functions (Graham, Cuoco, & Zimmermann, 2010, p. 41). The researcher chose, modified and made activities to engage students and allow them to discover and 'reinvent' (Gravemeijer & Doorman, 1999) some mathematical ideas themselves. This was so that the learning experience would increase students' engagement and confidence in mathematics.

1.3 Implementation, Research Methodology and Research Methods

The 24 participants were recruited from the researcher's established Transition Year mathematics class which is in a mixed post-primary school. In Transition Year in this school there are two mixed-ability classes for students who will sit either the Leaving Certificate Ordinary Level or Foundation Level paper. These two class groups would represent 40% of students within the year group. The other 60% of the year group intend to sit the Leaving Certificate Higher Level paper. The students were aged between 15 and 17 years.

The duration of the learning experience was thirteen 40 minute class periods, some of which were double-periods of 80 minutes. The learning experience took place in the school's computer room. The participants were divided into teams of four based on including a 'more knowledgeable other' in each team and also on the basis of which individuals would work well together.

The research methodology used was an exploratory case study which is appropriate as it facilitates investigating a contemporary phenomenon in depth and within its real world context (Yin, 2013, p. 16). A classroom has many important contextual variables that could affect any learning experience. In order to understand the learning experience and to observe if there is any change in students' level of engagement it is important to capture as much of the experience in as deep a way as possible. A case study is suitable for this.

Yin (2013, p. 17) says case studies rely on many sources of evidence. A parallel mixed methods design (Creswell, 2008, p. 557) was used. Mixed methods were chosen to ensure that the richness of the learning experience was captured. Quantitative data on engagement and confidence was collected through administering the Mathematics and Technology Attitudes Scale (MTAS) (Pierce et al., 2007) prior to the learning experience and after the learning experience. To capture the depth and variety within the context of this technology-enhanced learning experience qualitative data was collected through researcher observation, through analysing the data captured by the website and through three semi-structured group interviews with a total of 11 participants lasting approximately 20 minutes each.

1.4 Findings and Analysis

It was found that there were statistically significant improvements in two areas – affective engagement and attitude to learning mathematics with technology. There were also indications in the evidence that there were some small positive changes in the areas of behavioural engagement, confidence with technology and affective engagement.

1.5 Document Roadmap

The next chapter details some issues in mathematics education, which include engagement and confidence in the subject. Why functions are an important area in mathematics and the variety of approaches for teaching them are described. Some student misconceptions in the area of functions are then outlined. The Realistic Mathematics Education (RME) approach to teaching is described as it is pedagogical theory that complements Bray's design heuristics. How technology can be used in mathematics education is then discussed. To convey the current trends in technology-enhanced mathematics education research a new classification created by Bray and Tangney (under review) is described. This classification is then added to by using the classification for three recent papers that are in the area of functions. Bray's design heuristics are then described.

The design of the experience is described in chapter 3. The chosen technology is described as well as the rationale for its choice. A justification as to why this learning experience should be considered a 'normal' learning experience, which can be replicated by others, is provided. A pilot of some of the activities and how this pilot informed the activities that were used in the learning experience is also described. The chapter finishes with a summary of how the design of the learning experience helps answer the research question.

Chapter 4 is the Methodology chapter. The chapter begins by outlining the research question. The implementation of the design and the research methodology is described and the rationale for the chosen methodology is provided. The research methods are outlined and a rationale for choosing them is explained. The data collection and data preparation techniques are also described. Ethical considerations are outlined and the limitations of the study are also discussed.

Chapter 5 presents the analysis of the findings from the quantitative and qualitative data. Chapter 6 summarises the findings, states the conclusions that can be drawn from the findings, discusses the generalisability and limitations of this study and suggests recommendations for future research.

2 Literature Review

This chapter details the area of research, the problems within the area, and a possible solution. It begins by outlining some issues in mathematics education, a justification for why functions are an important area of mathematics to research and the possible approaches for teaching functions. The misconceptions students have in the area of functions are outlined. The Realistic Mathematics Education (RME) approach to teaching is described as it is pedagogical theory that complements the principles of a set of design heuristics that are outlined later in the chapter. Following this the use of technology in mathematics education is outlined using a classification under four headings: type of digital tools, purpose of the activity, pedagogical foundations and levels of technology integration in the activity (Bray & Tangney, under review). Bray and Tangney's work is extended by applying their classification to three recent papers where technology is used to teach functions.

2.1 Issues in Mathematics Education

Mathematics is an important subject for individuals and society (Hoyles, 2016, p. 225) and knowledge and qualifications within it "are increasingly important gateways to further and higher education, for crucial life-skills and in order to respond to economic change" (Norris, 2012, p. 22). Given the importance of mathematics it is unfortunate that the subject is beset with numerous issues. These issues are the focus of this section.

Many of the issues in mathematics education centre on beliefs. Two types of beliefs were identified in the literature; beliefs about an individual learner's own ability with the subject and beliefs about the subject itself.

A learner can believe that they are poor at mathematics and this can impinge on their progress within the subject. If a learner believes they will not be successful in an area then they will have less motivation for the tasks in that area (Wigfield & Eccles, 1992). At the extreme end in terms of beliefs there are a significant number of people who suffer from "math anxiety", which is "an adverse emotional reaction to math or the prospect of doing math" (Maloney & Beilock, 2012, p. 404) and a meta-analysis by Ma (1999) found a relationship between anxiety toward mathematics and achievement in mathematics.

One could misunderstand that those with "math anxiety" are simply those with low ability level but this is not the case. Math anxiety is an impediment that is separate to an individual's mathematical ability (Maloney & Beilock, 2012, p. 404). A student with a certain level of ability and mathematical anxiety will not perform as well as a student with the same ability and no mathematical anxiety.

Some learners hold positive beliefs about mathematics. Many learners value the subject (Mullis, Martin, Foy, & Arora, 2012; UNESCO, 2012). This is important as students' beliefs about the usefulness of mathematics in terms of their future careers can affect their success in mathematics

(McNamara, 2013) and also their interest in mathematics provided they also have a high expectation of success (Star et al., 2014).

Negative beliefs about the subject can lead to issues in mathematics education. Unfortunately many students, even the students that value the subject, do not enjoy learning mathematics (Mullis et al., 2012). Leung (2006) explains that a positive attitude to mathematics is usually highly correlated with achievement. Others have found that many students with satisfactory results in mathematics dislike the subject and do not see why so much time should be spent studying it (UNESCO, 2012, p. 9).

It has been found that “most people of all ages regard mathematics simply as a set of procedures and rules” (Hoyles, 2016, p. 227) that is “often taught in skills-based, abstract, and decontextualized ways” (Watt et al., 2012, p. 14). Hoyles (2016) reports there are many who see the subject as boring and irrelevant.

Attitudes to the subject have a large bearing on achievement of students (Boaler, 1993; Kislenko et al., 2005; Ma, 1999) and research suggests that students who see the subject as boring will not engage with the subject nor increase their understanding of it (Boaler, 1993, p. 346; Kislenko et al., 2005). A curriculum can have great potential for engaging students in such an important subject but there is sometimes a gap between what is written (the intended curriculum) and what is experienced by students (the implemented curriculum) (Voogt & Pelgrum, 2005). While the intended curriculum can state that students will engage in activities, that they will work as mathematicians and think creatively, the implemented curriculum can often be a dry subject with an emphasis on only one way of getting the one correct answer. This gap between the intended curriculum and the implemented one can mean many students misunderstand what mathematics is. Hoyles (2016) describes how students “fail to glimpse in their efforts to master all the ‘machinery of the subject’, the key mathematical concepts, structures and relationships of the subject”.

There have been attempts to remedy this situation, both in Ireland and internationally, by overhauling curricula. These changes have attempted to include more metacognitive dimensions, such as creativity and problem-solving (NCCA, 2013b) within the learning of the subject, to embed mathematics within a meaningful context, and to focus more on conceptual understanding in tandem with skills development. Often these intended changes are misinterpreted and students end up finding it difficult to engage with what they see as a dry, skills-based and decontextualized subject (Watt et al., 2012).

This section has described the relationship between the beliefs about the subject and beliefs about an individual learner’s own ability within the subject and students achievement levels and engagement with the subject. With beliefs relating to the subject being important factors in students’ achievement levels, this research focusses on student engagement with, and confidence in, mathematics. The next section outlines the content area of mathematics in which this research is situated in, why this is an important content area of mathematics and the approaches used to teach the content area.

2.2 Functions: Their Importance and Approaches to Teaching them

2.2.1 The Importance of Functions

The content area that is the focus of this research is functions. "Functional thinking is analysing patterns (numeric and geometric) to identify change and to recognize the relationship between two sets of numbers" (Ontario, 2013, p. 8). Functions are included in all international curricula (Hodgen et al., 2010; NCCA, 2013b, 2013c; NCTM, 2000). The importance of functions is manifest in the reviewed literature but the topic has not always been regarded with such high esteem. Prior to the 1990's functions were primarily seen as a stepping stone to the important topic of calculus. It is only in recent decades that their importance in relation to algebraic understanding has been fully recognised (Kaput, 2000; Lagrange, 2014). Both the NCTM (2000) and Friel and Markworth (2009) emphasise the importance of understanding patterns, relations and finding rules to represent functions for algebraic understanding. Functions are now regarded as a topic that binds other topics together; playing a "central and unifying role" (Selden & Selden, 1992) within mathematics.

Before discussing approaches to teaching functions it is necessary to understand that functions can be represented in many ways, and can be set in many contexts. The various representations of functions are described, followed by a description of the types of contexts used for functions problems.

2.2.2 Representing Functions and Choice of Context

Functions can be represented in tabular form, graphically, symbolically, visually, verbally (Martin et al., 2009, p. 41), geometrically (Ontario, 2013), as ordered pairs (NCCA, 2013a) and as correspondences between two sets (Selden & Selden, 1992). A justification for using multiple representations to teach functions is that they can capture a learner's interest (Ainsworth, 1999). This should not be the sole reason for doing so. It is vitally important that students can move freely between many representations so as to have a full understanding of functions and it is also necessary for students to know which representation to choose in order to solve differing problems as some representations are more advantageous than others depending on the situation (Ontario, 2013).

Another facet that is important for any discussion of approaches to teaching functions is context. There are many choices about how one could use context when teaching functions. O'Keeffe and O'Donoghue (2011, p. 23) list four contexts in terms of types of problems: *Real*, *Realistic*, *Fantasy*, and *Purely Mathematical*. *Real* problems are created in a real environment while *Realistic* problems involve a simulation of reality. *Fantasy* problems have no basis in reality and are the product of imagination and *Purely Mathematical* problems relate only to mathematical objects.

Having outlined the importance of multiple representations for the teaching of functions and the choice of contexts that can be used for functions problems, the various approaches for teaching functions will now be explained.

2.2.3 Approaches to Teaching Functions

One could introduce the topic of functions using an *ordered pair* definition. Unsurprisingly, the research has found this approach is too abstract for pre-university students (Selden & Selden, 1992). However, Selden and Selden (1992) do note that abstraction should not be avoided and say that introducing new concepts using example after example is not advisable i.e. there is a necessity for definitions to further student understanding in mathematics.

A second approach is to show functions as *correspondences* between two sets. This approach is usually supported by means of drawing regions joined by arrows (Selden & Selden, 1992). An example of this is shown in Figure 2.1.

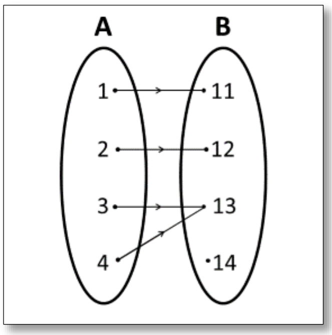


Figure 2.1 Example of correspondences between two sets

A third approach is to use graphs. These have advantages for interpreting some key features of functions, such as where a function is increasing, decreasing, and for identifying other key features of functions (Selden & Selden, 1992).

"Function machines" are usually presented as a picture with a space for an input, a crude image of a machine and a space for an output, as in Figure 2.2.

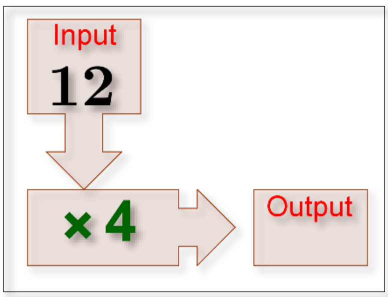


Figure 2.2 A Function Machine

The process students must apply to a variety of inputs is usually written on the machine. These instructions are usually a single processes e.g. "$\times 2$" or two processes e.g. "$\times 2 \quad +7$" and students are expected to apply the same process to a variety of inputs. An example of this is shown in Figure 2.3. A more demanding activity, that is sometimes, though not always, linked to function machines is where students are given lists of inputs and outputs and they must come up with the rule or process that links them, i.e. the students generalise the pattern. An example of generalising from lists of inputs is shown in Figure 2.4. This approach has seen positive results for students as they become more fluent in generalising patterns in unfamiliar situations (Nunes, Bryant, & Watson, 2009).

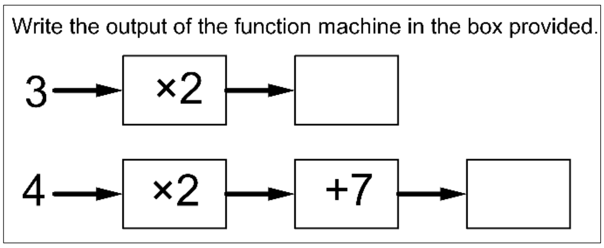


Figure 2.3 Function machines with a single process and a double process

x	y
1	5
2	6
3	7
4	8
5	9
6	…
7	11
8	…
…	…
…	

(i) When x is 2, what is y?

(ii) When x is 8, what is y?

(iii) When x is 800, what is y?

(iv) Describe in words how you would find y if you were told that x is ....

(iv) Use algebra to write a rule connecting x and y ....

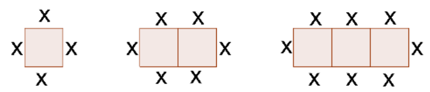
Figure 2.4 Generalising from a list of inputs and outputs (Nunes, Bryant, & Watson 2009)

Another method is to approach functions using solely algebraic symbols but this is less helpful for student understanding than using a variety of representations (Martin et al., 2009). Kaput (2000) concurs with this view and sees student experiences, where they memorise procedures to produce the correct string of symbols, driving many students away from mathematics. Given the issues with student engagement and confidence outlined in the previous section it is then not advisable to *solely* use purely mathematical context problems. Figure 2.5 is an example of a set of purely mathematical context questions.

If $f(x) = 3x - 2$, find (i) $f(10)$, (ii) the value of x for which $f(x) = 43$, (iii) $f(k)$, (iv) $f(2k)$

Figure 2.5 Purely Mathematical Context Questions

Another approach is to use geometric pattern tasks (Friel & Markworth, 2009), where the initial stimulus (or context) is a growing pattern. Figure 2.6 shows an example of a growing pattern task. This is usually a paper and pencil activity but it is also possible to use concrete materials such as pattern blocks (Chan, 2015), see Figure 2.7. These types of activities can facilitate combining the learning of functions with the learning of algebra (Friel & Markworth, 2009) and lend themselves to students exploring a pattern using other representations, such as tables and graphs, and can help students to generalise the relationship in symbolic form (see Figure 2.8). Crucially, these activities lend themselves to students understanding the purpose and usefulness of algebraic symbols and skills and even the advantages of the symbolic form (Ontario, 2013).



1. How many people can sit around 5 tables?
2. How many people can sit around 10 tables?
3. How many tables are needed for 56 people?
4. How many people can sit at n tables?
5. Express the relationship between the number of people who can sit at n tables as a function.
6. Make out a table with “Number of Tables” as the inputs and “Number of People” as the outputs.
7. Draw a graph of the table of values from the previous question.

Figure 2.6 A Growing Pattern Task (adapted from Friel & Markworth (2009))

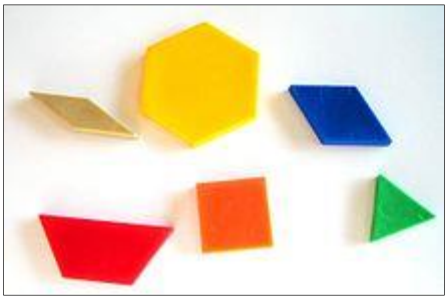


Figure 2.7 Pattern Blocks https://en.wikipedia.org/wiki/File:Plastic_pattern_blocks.JPG

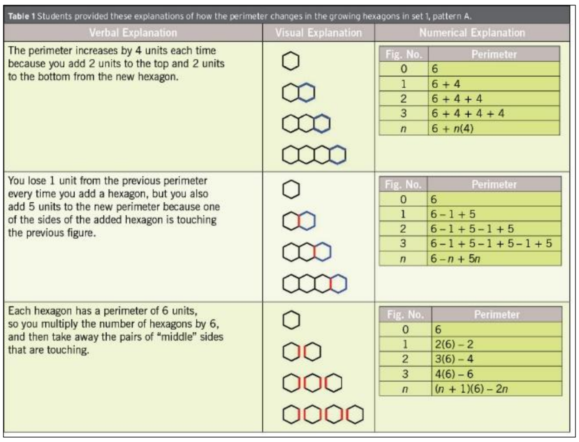


Table 1 Students provided these explanations of how the perimeter changes in the growing hexagons in set 1, pattern A.

Verbal Explanation	Visual Explanation	Numerical Explanation
The perimeter increases by 4 units each time because you add 2 units to the top and 2 units to the bottom from the new hexagon.		Fig. No. | Perimeter 0 | 6 1 | 6 + 4 2 | 6 + 4 + 4 3 | 6 + 4 + 4 + 4 *n* | 6 + *n*(4)
You lose 1 unit from the previous perimeter every time you add a hexagon, but you also add 5 units to the new perimeter because one of the sides of the added hexagon is touching the previous figure.		Fig. No. | Perimeter 0 | 6 1 | 6 – 1 + 5 2 | 6 – 1 + 5 – 1 + 5 3 | 6 – 1 + 5 – 1 + 5 – 1 + 5 *n* | 6 – *n* + 5*n*
Each hexagon has a perimeter of 6 units, so you multiply the number of hexagons by 6, and then take away the pairs of "middle" sides that are touching.		Fig. No. | Perimeter 0 | 6 1 | 2(6) – 2 2 | 3(6) – 4 3 | 4(6) – 6 *n* | (*n* + 1)(6) – 2*n*

Figure 2.8 Pattern Blocks Solutions (Chan, 2015)

Kaput (2000) also supports the idea of teaching functions and algebra concurrently. He combines this with the idea of using real-world contexts. He calls the real-word contexts he focusses on 'familiar quantities', and gives examples of quantities that change over time, e.g. heights of plants or people, temperature and numbers of people who are eating or asleep at various times throughout the day. He suggests it is useful to represent these contexts both pictorially and with time-based graphs.

The NCTM say that to develop reasoning and sense making with functions students should use multiple representations of functions, model situations with functions and analyse the effects of parameters when learning about each type of function (Graham et al., 2010, p. 41). Covariation is how one variable changes with respect to another and is another area of importance within functions as it is seen as essential for understanding concepts within calculus (Carlson, Jacobs, Coe, Larsen, & Hsu, 2002). The activities for the learning experience include all four types of thinking just mentioned i.e. multiple representations, modelling, analysing the effects of parameters and covariation.

2.2.4 Approaches that use a *Real* Context

Section 2.5 is Technology and Mathematics Education, but to complete the discussion of teaching approaches that began in the last section the approach of modelling a *Real* context is included here.

Through the use of technology other approaches to teaching functions have become possible. Technology makes it possible to collect data related to moving objects and creates the opportunity for students to explore *real* contexts. This can be done in a variety of ways. One method is to use a "Go!Motion" sensor, see Figure 2.9 below, to collect the position, velocity and acceleration data of moving objects. For example, the sensor can be used to record the position of a tennis ball thrown vertically upwards. This can lead to students working with expressions/functions for distance, velocity and acceleration which can be modelled using quadratic, linear and constant functions. Berry and Nyman (2003) used such a motion sensor and asked students to construct the original displacement-time functions from the graphs of a number of speed-time functions. The students were also asked to walk in a manner that would replicate the displacement-time graphs e.g. they walked slowly if the section of the graph that showed a small change in distance with respect to time and faster for sections that showed a large change in the distance with respect to time. They found that this approach helped students increase their conceptual understanding of functions through relating physical movement with graphical understanding.

Figure 2.9 Go!Motion Sensor

Another method that uses technology is to video a moving object and then use motion analysis software, such as Kinovea or Tracker, to record data relating to the position of the object. This data can then be analysed and students can model the *Real* context using their knowledge of functions and make predictions based on the model. Bray (2015) used technology in this way for two activities called Bungee Barbie and The Catapult Activity and found an increase in student confidence and engagement in mathematics as a result.

The previous subsections established the importance of functions, described multiple representations of functions, the many contexts of function problems and some possible approaches to teaching functions. The next section outlines some misconceptions students have in the area of functions.

2.3 Students' Misconceptions of Functions

Students have many misconceptions when dealing with functions and Carlson et al. (2005) describe nine such misconceptions. Not all of these are relevant to this study so only seven of them are described here.

1. Misunderstanding even the basics of inputs and outputs of functions.
2. Thinking that constant functions (e.g., $y = 5$) are not functions because they do not vary.
3. Viewing functions as two expressions separated by an equals sign. This is illustrated by (Thompson, 1994) as

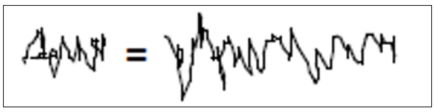


Figure 2.10 Concept image of a function (Thompson, 1994)

4. Having difficulty distinguishing between an algebraically defined function and an equation, e.g. Carlson et al. (2005) believe that students should work on activities “that promote students' thinking about an equation as a means of equating the output values of two functions, and the act of solving an equation as finding the input value(s) where the output values of these functions are equal”.

5. Thinking that all functions must “behave nicely” i.e. a function that is made of many pieces, a piecewise function, is not a function.

6. Distinguishing between visual characteristics of a situation and similar characteristics of the graph of a function that models the situation.

7. Difficulties in forming a function when given a function situation described in words. For example, university students with a high level of mathematical ability have difficulty modelling situations involving the rate of change of one variable with respect to another variable (Carlson et al., 2002).

These misconceptions are important to bear in mind when planning how to teach functions. An approach to teaching that encourages engagement in, and ownership of, learning in mathematics is described in the next section as an approach with these properties is advantageous for the design of a learning experience that aims to increase students’ engagement and confidence in mathematics.

2.4 Realistic Mathematics Education (RME)

As the research focus is concerned with engagement and confidence it is important to choose an approach that facilitates students to be engaged in the content of a learning experience and also one that shows that they are capable of constructing mathematical knowledge themselves.

RME is an approach for teaching mathematics that originated in Holland in the 1970’s. Hans Freudenthal was one of the principal leaders in the formation of the approach. In an earlier section four types of contexts were explained: Real, Realistic, Fantasy, and Purely Mathematical. With “realistic” in the name RME has often been misinterpreted to mean that the problem must be set in a Real or Realistic context. The “real” in RME comes from the Dutch expression “zich REALISEren” which means “to imagine” (Van den Heuvel-Panhuizen & Drijvers, 2014, p. 521). This means that problems set in any one of the four contexts are suitable for an RME approach. The crucial feature of the problems used in an RME approach is that the problems are experientially real, meaningful, and engaging in the students’ minds (Gravemeijer & Doorman, 1999; Knox, 2016; Van den Heuvel-Panhuizen & Drijvers, 2014, p. 521).

In RME, mathematics is not seen as a closed or ready-made system that should be learned (Freudenthal, 1971; Van den Heuvel-Panhuizen & Drijvers, 2014). Mathematics should be an activity where mathematical ideas are reinvented by the learner (Gravemeijer & Doorman, 1999). The learner is an active participant in the learning process, and engages in rich problems that require the learner to use and develop their own mathematical concepts. The problems should be designed so that the mathematics emerges from the problems. When learners apply their own mathematical knowledge to rich problems and where these problems involve real-life situations this is called horizontal mathematisation. Vertical mathematisation can also occur when the learner makes connections between concepts and strategies in the abstract domain of symbols (Van den Heuvel-Panhuizen & Drijvers, 2014, p. 522). Progressive mathematisation is the phrase used for when learners broaden and deepen their knowledge of mathematics both horizontally and vertically in an RME approach. As the learners are active in creating their own knowledge they feel a greater sense of independence, ownership and responsibility for the knowledge they themselves have constructed (Gravemeijer & Doorman, 1999, p. 116).

This section described RME as this approach was used in the design of the learning experience to increase students' engagement and confidence in mathematics. The next section addresses how technology can be used in mathematics education and the opportunities its use presents.

2.5 Technology and Mathematics Education

This section discusses how well technology is meeting its perceived potential to counter some of the issues discussed in an earlier section of this chapter and illustrate how technology should be used effectively in the teaching and learning of mathematics.

Technology has for a long time promised to alleviate some of the issues that beset mathematics education (Hoyles, 2016; Reed, Drijvers, & Kirschner, 2010). Hoyles (2016) sees the potential of technology to "reinvigorate engagement with mathematics" (p. 228). She acknowledges there are costs and other challenges in implementing technology in mathematics classrooms and sees these as the reasons why technology's potential impact has not reached its expectations (p. 234). As far back as 1992 Selden and Selden (1992) urged caution about "tacking" technology onto a busy syllabus as it could require substantial changes to a syllabus and could have the effect of aiding student understanding or create one more obstacle for students to contend with. Nunes *et. al.* (2009) are positive about the benefits of using technology but are a little defeatist when saying they feel the opportunities to take advantage of technology are not available in the normal school time.

Keeping these cautionary notes in mind the bulk of the literature is more positive about the inclusion of technology in mathematics education. Technology can aid in student understanding as it can deliver tools that are dynamic, graphical and interactive. This can enable learners to explore mathematical objects from multiple linked perspectives making the concepts easier to see, and (almost) tangible and manipulable (Hoyles, 2016, p. 231). Bardini *et. al.* (2004) are similarly positive about how technology allows students to work with real world problems through treating graphs as manipulable objects and explore solutions to problems using the numerous representations technology can portray. The idea of tangibility being important for student understanding of concepts in mathematics has been a key idea in educational literature since the work of Piaget and this idea is continued through in the learning theory of constructionism.

However, much of the first uses of technology in mathematics education was couched in the behaviourist learning theories of Skinner (Cooper, 1993). More recently there has been a shift away from using behaviourism towards using constructivism and social constructivism in more recent studies (Bray & Tangney, under review).

While using behaviourist strategies is less common in current technology-enhanced learning research, it must be said that using a behaviourist approach can work. Li and Ma (2010) cite a study where a behaviourist and a constructivist intervention was given to two different classes. Both classes improved greatly and exceeded normal achievement levels but the constructivist class did a lot better. While diverse ways of including technology can produce positive outcomes it is important to say that including technology does not automatically improve learning unless it is supported by good teaching and learning practice (Li & Ma, 2010).

With the content area of this study being functions it is important to note there is evidence that the affordances of technology can aid students understanding of functions (Nunes et al., 2009, p. 31):

> "We do not know if it is only in interactive computer environments that school students can develop a deep, flexible and applicable knowledge of functions, but we do know that the affordances of such ICT environments allow all students access to a wide variety of examples of functions, and gives them the exploratory power to see what these mean in relation to other representations and to see the effects on one of changing the other".

While the potential of technology may not be fully realised as yet the use of technology is included in the mathematics curricula of many countries (Geiger, Faragher, & Goos, 2010), including Ireland where both the Junior and Leaving Certificate syllabi include the use of "appropriate graphing technologies", such as graphing calculators and computer software (NCCA, 2013b, 2013c). In the United States technology is considered an essential component of the maths learning environment; influencing the mathematics that is taught and improving students' learning (Bu, Spector, & Haciomeroglu, 2011; NCTM, 2000).

With the focus of this study being engagement and confidence it is important to note that Mushi (2000, p. 29) found that learning mathematics through technology is interesting to students and it has a positive impact on students' attitudes towards the subject, while Star *et. al.* (2014, p. 3) found that there is a large body of literature on technology and motivation and that the effectiveness of technology as a motivational tool are mixed. They put forward that some of this may be due to the naïve use of technology as a "secret sauce" that automatically increases engagement. It is important to base the use of technology in principles gleaned from the research in the area, as this study aims to do.

This section has broadly described how technology can be used in mathematics education. The next section describes a system for classifying the technology interventions in mathematics education research.

2.6 Bray and Tangney's Classification of Mathematics Education Research

The previous section summarised and synthesised the relevant literature for using technology in the teaching and learning of mathematics. This section describes an up-to-date system for classifying the recent studies of technology interventions in mathematics education research (Bray & Tangney, under review). The classification is under four headings: Technology, Learning Theory, SAMR Level and Purpose. The components of the classification are shown in Figure 2.11. The headings of Technology, Learning Theory and Purpose are self-explanatory, however SAMR Level describes the level of technology adoption for a task or activity.

Technology	**Learning Theory**	**SAMR Level**	**Purpose**
Collaborative by Design	Behaviourist	Substitution	Change in Attitude
Dynamic Geometry Environment	Cognitive	Augmentation	Improve Performance
Multiple Linked Representations	Constructivist	Modification	Improve Conceptual Understanding
Outsourcing – Computation	Social Constructivist	Redefinition	Skills-focused
Outsourcing – Content	Constructionist		Support Teachers
Programming Tools			Collaboration and Discussion
Toolkit			

Figure 2.11 Components of the Classification (Bray & Tangney, under review)

The classification developed from the analysis of 139 recent research papers and built upon an initial analysis of 25 papers in 2012 (Bray & Tangney, 2013). By creating the classification the authors have produced a way of discussing the recent research in technology-enhanced mathematics education under four headings. Each of these headings are now be described with any relevant theme from current research included.

2.6.1 Technology

The first category of the classification is Technology. Using technology in mathematics education can vary widely in terms of cost and complexity from using repurposed television program clips to immersive virtual reality environments (Star et al., 2014, p. 1). Bray and Tangney's category of Technology significantly extends the work of the two classifications of technology created by Hoyles and Noss (Hoyles & Noss, 2003, 2009) and differentiates technology into seven components, *Collaborative by Design*[3], *Dynamic Geometry Environment (DGE)*[4], *Multiple Linked Representations*[5], *Outsourcing – Computation*[6], *Outsourcing – Content*[7], *Programming Tools*[8] and *Toolkit*[9]. Outsourcing the delivery of content was the most prevalent type of technology used in the papers analysed. 27% of the studies used this type of technology. 24% of papers used DGEs and 19% of papers outsourced computation.

2.6.2 Learning Theory

The second category of the classification is Learning Theory, which is divided into five components. It includes Behaviourist from the work of Skinner and Cognitivism from the work of Bruner. Constructivism developed from cognitivism and constructivism has two further branches, Social Constructivism and Constructionism. This means the Learning Theory category has five components: *Behaviourist*, *Cognitive*, *Constructivist*, *Social Constructivist* and *Constructionist*.

One theme revealed in the analysis of recent research is the predominance of the constructivist (37%) and social constructivist (34%) learning theories compared with recent research based on behaviourist learning theory, which is almost absent (2%).

2.6.3 SAMR Level

The third category of the classification, SAMR Level, deals with level of technology adoption for a task or activity. It is based on the model produced by Puentedura (2006). This model is shown in Figure 2.12.

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[3] Collaborative by design describes technologies that increase connectivity between learners, for example online forums.

[4] Dynamic Geometry Environments (DGE) describe software that allows the user to create and manipulate geometric constructions.

[5] Multiple Linked Representations (MLR) link many representations of single mathematical entities e.g. functions, and can also, for example, combine a DGE and Computer Algebra System (CAS).

[6] Outsourcing – Computation describes technologies, such as Computer Algebra Systems (CAS) and graphics calculators, that can allow students to manipulate mathematical objects e.g. equations, graphs and points swiftly to allow student exploration.

7 Outsourcing – Content describes technologies that deliver content that would traditionally have been delivered by a teacher.

8 Programming tools provide novel ways of modelling and representing mathematics.

9 A Toolkit describes technologies that are designed in accordance with a specific pedagogical approach, that provide support for the student and the teacher through tasks and lesson plans, and provide feedback for assessment (Bray & Tangney, under review, p. 10).

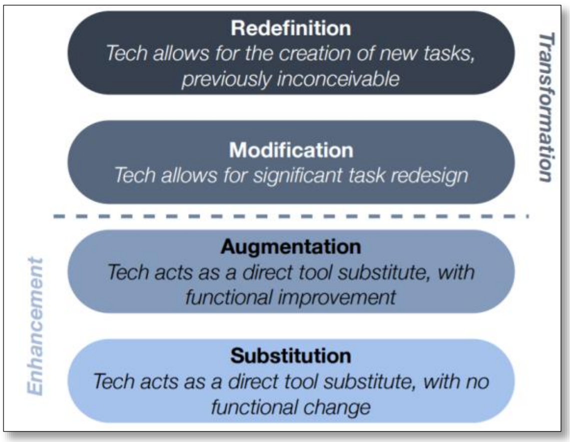

Figure 2.12 The SAMR Model (R. Puentedura, 2012)

Puentedura's model first divides the use of technology for learning activities into two sections, Enhancement and Transformation, Figure 2.12. Each of these sections is then subdivided into two levels. The first level of Enhancement includes *Substitution*, which is using technology as a replacement for traditional tools with no added functionality gained e.g. replacing a textbook with an eBook that has no interactive features. The second level of Enhancement is called *Augmentation*, which is using technology in a style that is similar to what would be done without the technology but with some functional improvement e.g. computer-based tests that are corrected more quickly than could be achieved by a teacher.

The Transformation section has two levels and is relevant to mathematics as technology can change what content in mathematics can be explored and how traditional mathematics should be learned (Bu et al., 2011). The first level is *Modification* and describes learning experiences that have been significantly redesigned using the capabilities of the technology e.g. using a Dynamic Geometry Environment to graph a multitude of functions to discover the key features of different function types. The second level is *Redefinition* and includes learning experiences that would not be possible without the affordances of technology e.g. using an online forum to facilitate collaborative problem solving asynchronously.

Bray and Tangney's review found no papers reporting the use of technology in a substitutive manner. It is likely that technology is being used in this way in many classrooms, but this type of usage is not being researched. Despite the calls for the transformative use of technology in the research (Geiger et al., 2010; Hoyles, 2016) the majority of the interventions, 61%, were classified at the Augmentation level. The remaining 39% were classified at the Transformative level; 25% at the Modification level and only 14% at the Redefinition level.

2.6.4 Purpose

The fourth category in the classification is Purpose and classifies the primary purpose of a research paper into components called *Change in Attitude*, *Improved Performance*, *Development of Conceptual*

Understanding, Skills-focused, Support Teachers and lastly *Collaboration and Discussion*. Many of the purposes do not require explanation but the authors do explain that *Change in Attitude* incorporates motivation, self-efficacy and engagement. *Improved Performance* tended to be studied using pre- and post- tests and would be concerned with content knowledge and should not be confused with *Skills-focussed,* which covers collaboration, problem-solving and creativity. It is worth noting that some of the interventions in the analysed papers had to be classed as having more than one purpose as the aims of the interventions were inextricably linked.

2.6.5 Summary

The components of each of the four categories of the classification and the relevant themes from the analysis of the papers were described above. Given the vast array of choices of combinations within the four categories of the classification it is critical to know what is effective when integrating technologies in schools so it is worth reiterating the finding by Bray and Tangney (under review) that combining a "constructivist, team-based, project-based pedagogic approach, and non-standardised assessment methods produces the greatest positive effects (Ertmer & Ottenbreit-Leftwich, 2010; Li & Ma, 2010; Voogt & Pelgrum)". The design chapter presents how these findings are implemented in the learning experience for this study. The next section draws together some of the ideas set out so far by showing how the classification described above can be applied to three recent papers on using technology to teach functions.

2.7 The Classification Applied to Research Papers Using ICT to Teach Functions

This section illustrates how the classification described above can be applied to some recent papers. These papers were not analysed by Bray and Tangney, so this section extends their work. This section classifies three papers that used technology to teach functions.

The papers were found through searching the online databases of the Trinity College Library, and Google Scholar with the search terms "math education function digital", "ICT function math" "math education technology". To obtain some up to date research papers email alerts from Google Scholar were created with the same search terms as above.

The first paper to be classified is one by Lindenbauer (2017). It examines how using GeoGebra worksheets can have an impact on students understanding of functions. 28 students completed a diagnostic test and then worked in pairs for three lessons on tasks that used GeoGebra applets.

	Classification	**Rationale**
Technology	DGE	The technology used is GeoGebra, which is a Dynamic Geometry Environment.

Learning Theory	Cognitive	The focus was on influencing students' "conceptions and internal representations" of aspects of functions (Lindenbauer, 2017, p. 1).
SAMR Level	Redefinition	The example applet in the paper involves students adjusting a time slider to see how far a billiard ball is from the side of a billiard table at various times and includes the use of a "Dynagraph" which has two parallel axes for time and distance from the side of the table. This type of activity would not be possible with traditional technologies, such as pen and paper.
Purpose	Conceptual Understanding	The research is investigating what influence dynamic applets have on students understanding of functions and how dynamic applets should be designed to support students' conceptual understanding of functions.

The second paper to be classified is by Kissi, Opoku and Boateng (2016). They investigated if the Microsoft Math Tool could be used to increase student understanding of linear functions. Two groups of similar abilities learned about linear functions. One group used technology and other one did not use technology.

	Classification	**Rationale**
Technology	DGE	The students used the software to see linear functions in many representations.
Learning Theory	Constructivist	The activities were designed so the students could generate their own knowledge themselves and not through knowledge transfer from the teacher (p. 119).
SAMR Level	Augmentation	The activities could have been conducted using traditional pencil and paper methods. The technology increased the speed at which the discoveries about the properties of linear functions were made.
Purpose	Conceptual Understanding	Two groups were chosen. One group used technology and the other did not use technology. The research investigated if there was a difference between the performances of the two groups on a test of understanding of linear functions.

The third paper to be classified is by De Almeida, Gomes, Spinillo, and Saraiva (2016). They investigated the use of a blended learning approach to increase students' problem-solving strategies for linear functions.

	Classification	Rationale
Technology	Collaborative by Design	An 'Educational Social Network' called REDU, which promotes "collaboration and communication in education" (p. 846) was used.
Learning Theory	Social Constructivist	Asynchronous collaboration was used and students learned from each other. Questions that would normally be asked of and answered by a teacher were answered by other students.
SAMR Level	Redefinition	The technology facilitated asynchronous collaboration between the students that would not have been possible without the technology.
Purpose	Skills-focussed	The purpose was to increase students' problem-solving strategies for linear functions.

The number of papers classified above is small. It is not surprising the papers had DGE's as the technology for facilitating the learning of functions as many are designed for this purpose. It would be unwise to draw any conclusions from just three papers. The papers do fit into the findings of Bray and Tangney in the following ways:

- No paper was at the Substitution level of the SAMR model.
- No paper were based on the behaviourist learning theory

2.8 Bray's Design Heuristics

In an earlier section of this chapter the multitude of issues impinging on mathematics education were outlined. The affective domain, e.g. confidence of a learner, is part of this and can impact on a learner's willingness to fully engage with the subject of mathematics. Bray (2015) recognised these issues and devised a set of design heuristics which combine the findings in the literature about good practice regarding choice of pedagogy, usage of technology and mathematics education and 21st century learning and activity design to mitigate against such problems (Knox, 2016). The heuristics are a "set of desirable attributes of technology-mediated mathematics learning activities that have the potential to increase student engagement and confidence" (Bray, 2015, p. 164).

Bray developed the heuristics through applying them to a variety of topics including statistics and probability, geometry and trigonometry, number, and functions (Bray, 2015, p. 9). Bray had two phases to her research. The interventions in the first phase took place in the Bridge21 Learning Laboratory. The students involved were all familiar with the Bridge21 lesson activity structure and had all volunteered to take part in the interventions. During the second phase the four interventions took place in school settings. The students involved were all familiar with the Bridge21 lesson activity structure and had not volunteered to take part in the interventions. The in-school interventions did not fit into the confines of the normal school timetable, which commonly consists of 40 minute or 80 minute periods. They interventions were for (i) two hours a day for a week, (ii) two days from 10am to 4pm each day, (iii) two hours in a single afternoon and (iv) two hours a day for a week.

The five parts of the heuristics are:

"1. Activities should be team-based and encourage collaboration, in accordance with a socially constructivist approach to learning.

2. Activities should exploit the transformative as well as the computational capabilities of the technology.

3. Activities should make use of a variety of technologies (digital and traditional) suited to the task, in particular, non-specialist technology such as mobile phones and digital cameras that students have to hand.

4. Tasks should:

- involve problem-solving, investigation and sense-making,
- involve guided discovery,
- be situated in a meaningful/real context,
- move from concrete to abstract concepts,
- be open-ended but with constraints,
- be cross-curricular/cross-strand,
- be focused on skill development as well as on content,
- have a 'low-floor' and a 'high-ceiling'.

5. Activities should be structured in accordance with the Bridge21 model (or a suitable structured alternative) of 21st Century Learning and activity design" (Bray, 2015, p. 164).

The Bridge21 model was the model of 21st Century Learning chosen. It was the model used by Bray when she applied the heuristics and has been used successfully for creating innovative learning experiences in many areas, such as mathematics (Tangney, Bray, & Oldham, 2015), physics (Girvan, Wickham, & Tangney, 2016) and others.

The five heuristics are the pillars upon which the learning experience was designed. The design of the learning experience will be described in the next chapter.

2.9 Summary

This chapter detailed the area of research, the problems within the area, and the basis for a possible solution. It began by outlining some issues in mathematics education, a justification for why functions is an important area of mathematics to research and the possible approaches for teaching functions. The misconceptions students have in the area of functions were described. RME was described as it is a pedagogical theory that complements the principles of Bray's design heuristics. Following this the current trends in the use of technology in mathematics education were outlined using a classification under four headings: type of digital tools, purpose of the activity, pedagogical foundations and levels of technology integration in the activity (Bray & Tangney, under review). Bray and Tangney's work was extended by applying the classification to three recent papers where technology was used to teach functions.

Bray's set of design heuristics have been chosen as the basis for designing a rich learning experience in the topic of functions to improve student engagement and confidence as they have yielded positive results in these areas. The focus for this research comes from one of Bray's own recommendations for future research. She recommended that to extend her research more in-school interventions could be undertaken to see if the results are sustainable (2015, p. 172). In particular, she suggested applying the heuristics for a greater number of classes with similar students and using content that is aligned to syllabi that are examined in state examinations. (As will be described later the content for the learning experience for this study is all curriculum aligned to the content for the Leaving Certificate Ordinary Level syllabus, which is examined in state examinations and the learning experience took place in a normal school setting.) Bray's interventions were conducted outside of the confines of a normal school timetable, which commonly consists of 40 minute or 80 minute periods. This study builds upon Bray's work as the heuristics were given a robust test within the confines of a normal school timetable. The research question is can participation in a rich learning experience, designed in line with Bray's heuristics, improve student engagement and confidence when applied to the topic of functions within the confines of a normal school timetable?

The next chapter provides an account of how the technology-enhanced learning experience incorporates key themes outlined in the literature presented in this chapter.

3 Design

3.1 Introduction

The previous chapter outlined how problems with engagement with, and confidence in, mathematics can affect students' achievement levels in the subject. Bray's heuristics, which have the potential to increase student engagement with, and confidence in, mathematics (Bray, 2015, p. 164) and the complementary teaching approach of RME were described. This research will focusses on how participation in a rich learning experience, designed in line with Bray's heuristics, improves student engagement and confidence when applied to the topic of functions within the confines of a normal school timetable. There were many design decisions made to create the technology-enhanced learning experience for this study. How these were informed by the key findings in the review of the literature will be outlined in this chapter.

The remainder of the chapter begins by describing the criteria used for choosing the technology for the learning experience. The chosen technology is then described followed by why the technology should be considered a *Toolkit*. This is followed by an outline of the learning experience and how it reifies Bray's heuristics. The learning experience is then presented as a 'normal' learning experience, which could be replicated by others.

A description of a pilot of a small number of activities and a rationale for the length of the learning experience is also included. Subsequently, an in depth analysis of how each activity relates to each point of the heuristics is outlined through tables that describe each activity, the type of functional thinking the activity focuses on, the curriculum content the activity is focused on and the how the activity relates to the identified challenges students have in learning functions. The penultimate section of the chapter describes how this research is classified under each of the four headings of Bray and Tangney's classification of mathematics education research (Bray & Tangney, under review). The chapter is then summarised.

3.2 Choice of Technology

A number of technologies were considered for the learning experience. The ability to create activities based on big ideas for developing understanding of functions that were found in the literature - using multiple representations, covariation, modelling and analysing the effects of parameters (Carlson et al., 2002; Graham et al., 2010) – was vitally important. Additional criteria for selecting a suitable technology or combination of technologies for the learning experience were being able to collect data about students' work – including written understanding of some aspect of activities – to check progress, ease of use for students, and being able to facilitate the creation of activities that engage students in the topic of functions. A reason for including written understanding of some aspect of the activities was that students can find questions that require explanations quite challenging and this was one way of ensuring the level of challenge was high enough to encourage collaboration within the teams.

A computer-based graphing calculator or a DGE were the technologies considered for the learning experience as both technologies provide the space for students to explore and learn about functions using multiple representations. The researcher has extensive knowledge of DGEs, especially GeoGebra. GeoGebra was strongly considered for the learning experience as it could have provided the space for students to explore and learn about functions using multiple representations, allows students to modify parameters of functions easily and see the effects of these modifications. GeoGebra could also have been used for modelling situations with functions and the technology also includes the facility for sketching freehand functions on the screen, which is useful for covariation problems.

The researcher previously had students using GeoGebra to explore geometry concepts by building artefacts from scratch and found it took students quite a number of classes for students to become adept at using it. The current research involves a short intervention, with a duration of 8 hours so having students spending time on becoming accustomed to the software so that they could build artefacts from scratch was dismissed. A second approach using GeoGebra was to have students manipulate and/or add to premade artefacts. This would have lessened the burden of becoming adept at GeoGebra, but these premade files would have to be stored somewhere that was easily accessible e.g. a small website. The other criteria, easy collection of data and being able to create engaging activities, were then considered. To be able to collect the results of students' interactions with a DGE would have required the students to save the file and email it to the teacher or to upload it to an online forum or a website, which is cumbersome. To collect students' written understanding of some aspect of activities with a DGE would have required students navigating away from the DGE to use some quiz software, a forum or even to use pen and paper which would have also been cumbersome. Creating engaging activities with GeoGebra was also possible but if a video was to be included, this would have meant navigating away from GeoGebra, which would not have been ideal. In summary, a DGE, such as GeoGebra, could have been used for the learning experience but it would have necessitated the building a small website and also using some quiz software.

The researcher knew of web-based software called Activity Builder, which is based on the powerful online graphing calculator produced by Desmos. This graphing calculator can use multiple representations, allow students to modify parameters of functions to see the effects of these modifications and can also be used to model situations with functions. Activity Builder adds greater functionality to the online graphing calculator, one of which is free hand sketches, which is useful for covariation problems.

Activity Builder fulfils the criterion of being able to collect data of students; both the results of students' interactions with different representations and written understanding of some aspect of activities.

Activity Builder is also a suitable technology in regards to ease of use for students (Thomas, 2015). It has a low floor; meaning it is easy to get started with the technology and it has a high ceiling in that the software is capable of graphing functions much more complex than are used in this learning experience. Activity Builder also fulfils the final criterion as it can be used to create engaging activities in the topic functions, through including videos, animations and games. Having described how Activity Builder satisfies the criteria required greater detail will now be given on the software.

Desmos produced this popular web-based tool for creating mathematical activities. The website contains publicly available activities which were created by Desmos and other activities created and shared by individual teachers. Many more activities have been created by individual teachers and are used privately. Desmos released their first activity, Penny Circle, in November 2012 and made it possible for teachers to create their own activities when they released the Activity Builder on July 31, 2015.

At Activity Builder's core is the powerful graphing calculator that Desmos are known for. Desmos describe Activity Builder as "a sequence of screens, each with a different task, prompt, or question". Images and videos can be included in the screens (Figure 3.1). Students can type and manipulate mathematical objects (e.g. points, or functions) and these are displayed on the screen. Students can be given many tasks to check their understanding, such as plotting points, typing functions (Figure 3.2), matching representations, sketching freehand graphs (Figure 3.3), answering multiple choice questions, completing tables of values (Figure 3.4), manipulating mathematical objects (Figure 3.5) or typing their understanding of a question (Figure 3.6).

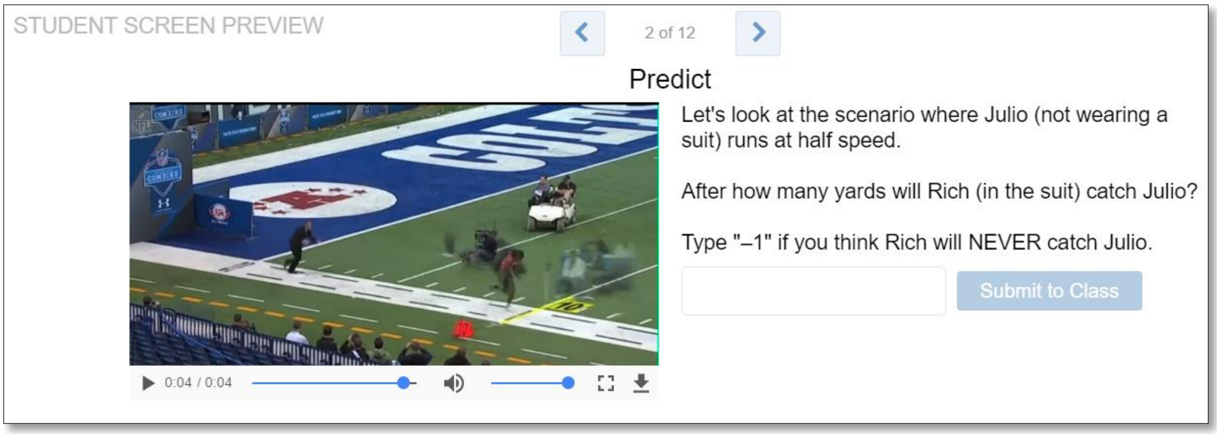

Figure 3.1 Video capability within activities

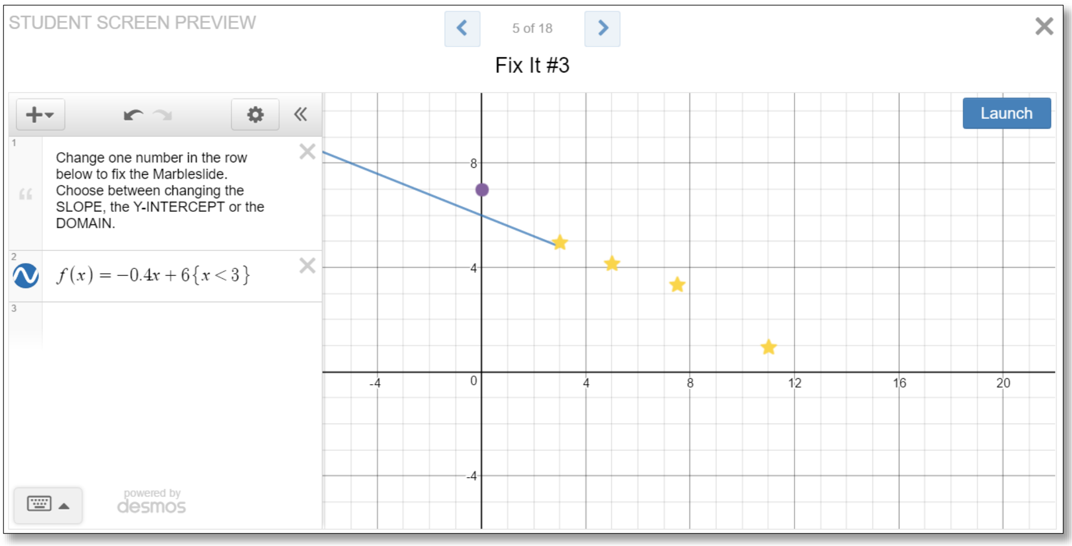

Figure 3.2 Typing Functions

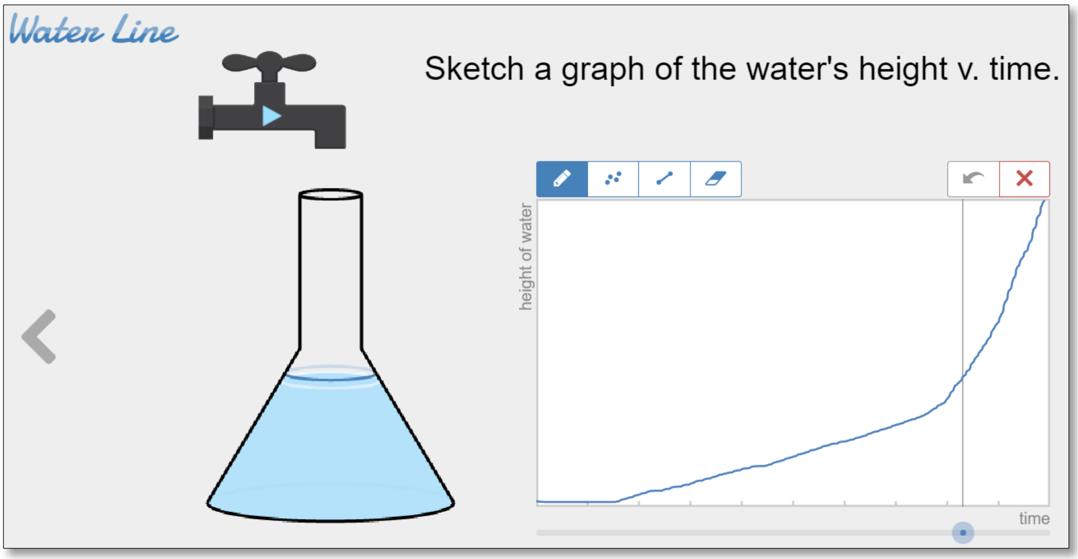

Figure 3.3 Sketching Freehand Graphs

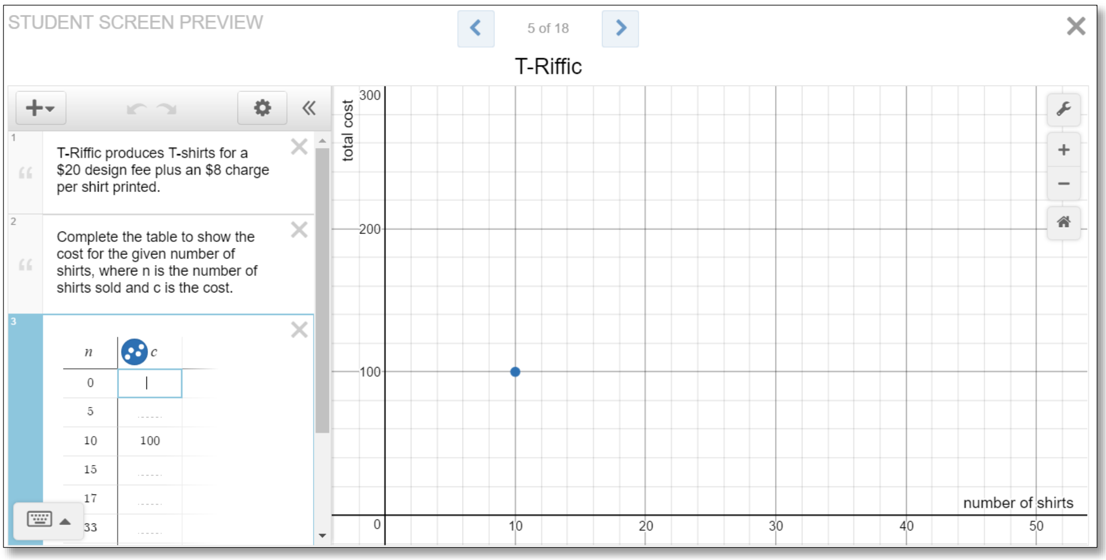

Figure 3.4 Completing a Table of Values

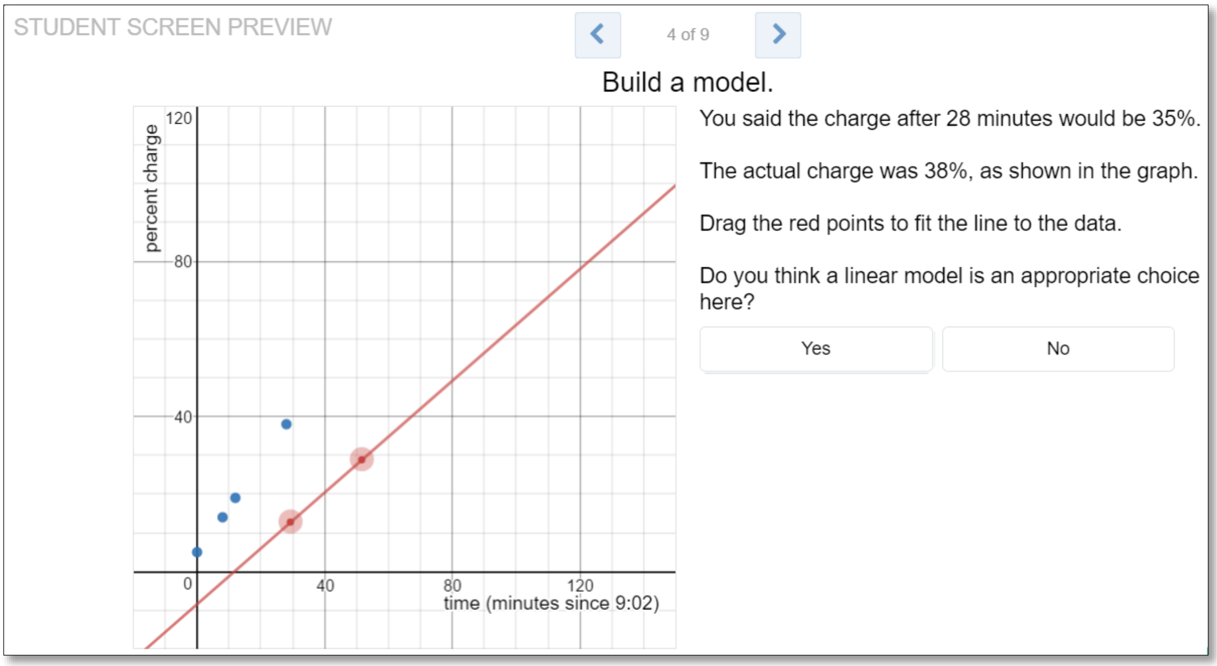

Figure 3.5 Manipulating Mathematical Objects

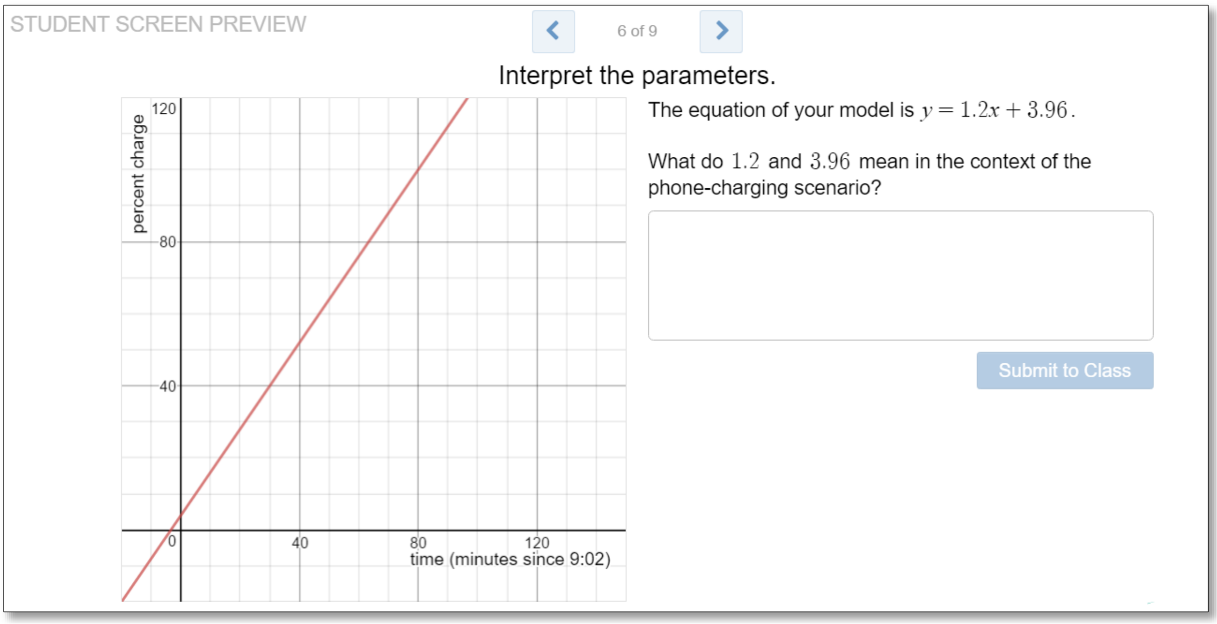

Figure 3.6 A question that requires students to type their understanding

An advanced feature, called a "Lab" of Activity Builder include the option to include create levels of an engaging game called Marbleslides (Figure 3.7). This game requires students to understand the key features of the type of function that is the focus of the lesson e.g. linear, quadratic, trigonometric etc. By changing the parameters of a function (e.g. domain, slope, *y*-intercept) the students can make marbles that slide on top of the function hit all the stars on the screen. Sometimes it is essential to create new functions so the marbles hit all the stars.

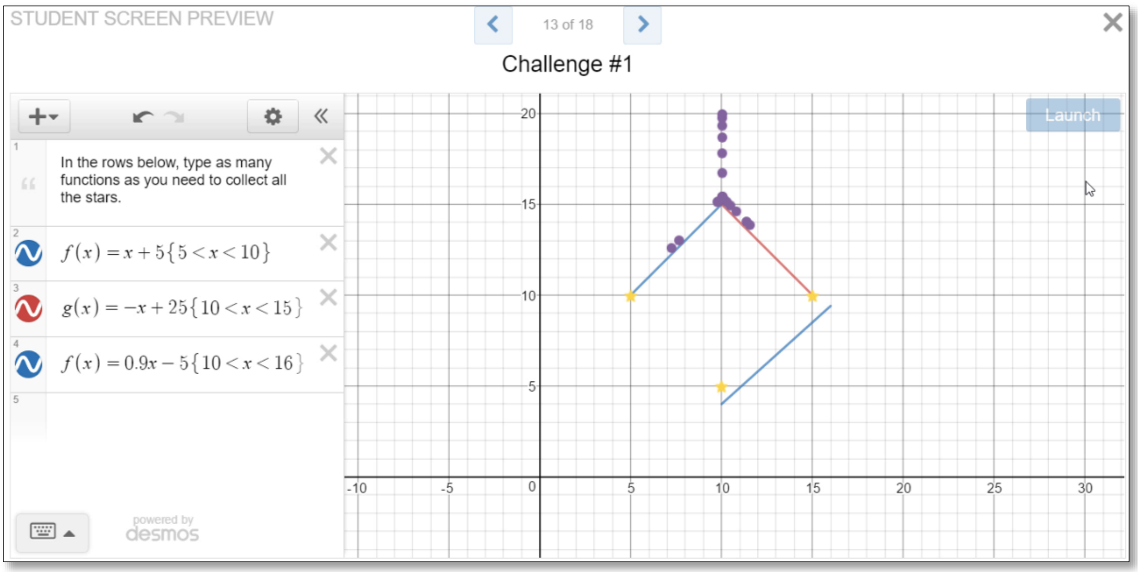

Figure 3.7 Marbleslides

Activity Builder can also be used to create games called Polygraphs (Figure 3.8). A Polygraph is a game similar to the children's game Guess Who. Teams identify a graph chosen by another team by asking questions that can only be answered Yes or No. The questions should relate to the properties of linear functions i.e. slope, y-intercept and domain. The purpose of Polygraph activities is to develop greater precision in language e.g. move from "Is the function going up?" to "Does the function have a positive slope?".

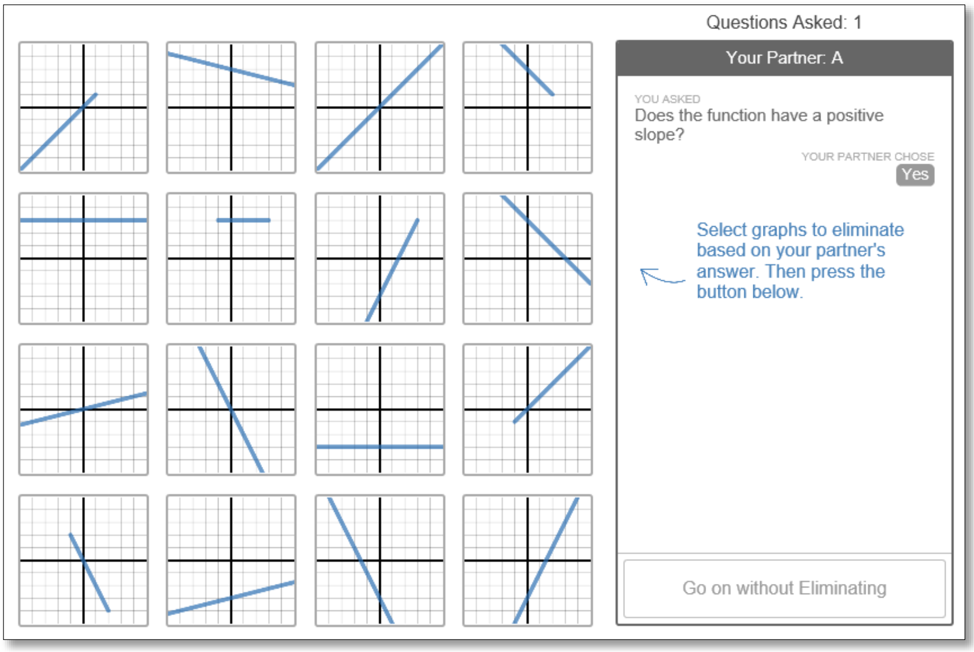

Figure 3.8 A Polygraph Activity

Greater detail on the variety of data Activity Builder collects for checking student understanding will now be described. The tool is web-based and once each screen of an activity is completed (through sketching, manipulating mathematical objects, or typing etc.) the data is automatically saved. There is no need for students to email their work which makes data collection easy for a researcher and a teacher. This data can be analysed by the teacher at any time. This is an example of what Hoyles (2016, p. 229) terms " 'invisible' formative assessment with data collected as students work on their solutions (individually or collectively)". The way the software integrates opportunities for students to verbalise their thoughts on various screens without the need for recording answers using pen and paper or an additional technology such as a forum is very useful. The variety of data that can be collected was seen as invaluable from the point of view of analysing if there was evidence of learning throughout the learning experience.

Activity Builder is able to package content in a visually appealing and engaging way, with its sequence of screens that can include video, images and (in some of the activities created by Desmos) animations. This variety allows for the creation of engaging content for students. Each screen can have a different starting point or stimulus and the students can be challenged to use differing mathematical processes on later screens. This enables the mathematical content to be examined from many points of view and to broaden and deepen students' understanding of concepts i.e. progressive mathematisation. This variety of starting points also keeps the level of challenge and level of engagement at a high level.

Having explained why the technology was chosen the next section will present why the technology should be considered a *Toolkit*, which is one of the components within the categories of Technology in Bray and Tangney's (under review) classification of mathematics education research.

3.3 Activity Builder as a *Toolkit*

Having explained why Activity Builder was the chosen technology an explanation of why Activity Builder should be considered a *Toolkit* now follows. Bray and Tangney (under review) describe a *Toolkit* as:

> "the design of technologies in accordance with a specific pedagogical approach, along with the provision of support for the student and the teacher through tasks and lesson plans, and feedback for assessment, all founded in the relevant didactic theory."

The following paragraphs justify why the software is a *Toolkit* by looking at 1. the specific pedagogical approach 2. the support provided and 3. the feedback for assessment.

For a technology to be considered a *Toolkit* it must be situated within a pedagogical approach. The Desmos website describe their view of the pedagogy of Activity Builder in a post on their blog entitled "The Desmos Guide to Building Great (Digital) Math Activities". Appendix 1 has the full text of this post, but the headings of some of their paragraphs illustrate how this software is appropriate for designing a technology-enhanced learning experience based on social constructivism. They believe the students are the ones who should create their own learning – "Give students opportunities to be right and wrong in different, interesting ways". They encourage a minimal amount of instruction – "Keep expository screens short, focused and connected to existing student thinking". They see the teacher as a facilitator in the creation of knowledge – "Create objects that promote mathematical conversations between teachers and students". In a separate blog post Desmos explain how they delayed including video and multiple choice questions in Activity Builder as they wanted to build these features in a way that was aligned to their pedagogical principles (Desmos, 2016c). For example, the default format for multiple choice questions requires an explanation after the choice has been made and they say this took them time to design and code in line with their "pedagogical preferences" (Desmos, 2016c). They go on to explain how they were interested in creating a "house style" that engaged students through informal mathematical thinking, which they believe makes formal mathematical thinking easier to learn. This would resonate with students constructing their own knowledge and also resonates with the progressive mathematisation aspect of RME that was outlined in the review of the literature. Based on a review of the what Desmos say and from previewing and using a number of their activities it is possible to conclude the underlying pedagogical approach for Activity Builder is a socially constructivist one. This fulfils the specific pedagogical approach aspect of the definition of a *Toolkit*.

Activity Builder also satisfies the support criterion of the description of a *Toolkit* as the website provides a huge amount of support to teachers. There are a large number of activities available on the website and each activity is accompanied by a detailed description of the purpose of the activity. Additional support is provided to the teacher when previewing an activity through "Tips for Teachers". This level of description for each activity and the suggested duration of each activity being equivalent to a full lesson would mean that the website is providing what equates to lesson plans in the definition of a *Toolkit*. Desmos have created numerous engaging activities that could be considered meaningful activities for students. The software facilitates the creation of other meaningful activities that can be tailored to the specific curriculum, prior knowledge and ability level of the intended users of the activities. The Desmos website also provides support through what it terms "bundles". These are described as "a collection of activities chosen and sequenced (with love) by the Desmos Teaching Faculty" (Desmos, 2016a). All of this fulfils the support aspect of the definition of a *Toolkit*.

Feedback for assessment is the third criterion for a technology to be considered a *Toolkit*. This feedback is provided through a Teacher Dashboard. The progress of students can be tracked, and it is easy to analyse their typed responses or the graphs they have plotted or sketched. The dashboard allows the teacher to see an individual student's attempt at all the tasks or to see each students' attempts at an individual task. This fulfils the feedback for assessment aspect of the definition of a *Toolkit*.

Activity Builder should be considered a *Toolkit* as it satisfies the three criteria of the definition. The next section looks at the learning experience.

3.4 The Learning Experience

3.4.1 Introduction

The previous two sections described the technology, why it was chosen, and why it should be considered a *Toolkit*. This section describes the learning experience. The principal factors in the design of the learning experience were Bray's heuristics. To build on Bray's work the learning experience was implemented in a 'normal' setting as opposed to a 'special' setting. The section begins by distinguishing between 'normal' and 'special' interventions. There is then a description of how Bray's heuristics were applied in the design of the experience and some detail on a pilot of some of the activities and the reasons for the length of the intervention.

3.4.2 A 'Normal' Learning Experience

Some technology interventions could be considered special interventions, or boutique interventions (Tangney et al., 2015). Interventions such as these might include a small number of participants, or the participants might be volunteers, there may be a greater number of teachers than is the norm or could take place outside of the normal school setting. Such interventions are valuable to further the understanding in various fields of research but they suffer from being less likely to be replicated by others for the purposes of teaching and learning as opposed to research purposes. To ensure the learning experience could be replicated by others the learning experience was designed within the constraints of a normal classroom and school setting and would only use resources that are commonplace and normally available.

A strength of this current research is that this learning experience is not a special intervention, and consequently could be easily replicated by others. The reasons it should not be considered a special intervention are that it was operated:

- with an established class group i.e. the participants were not volunteers (ethical procedures are outlined in Chapter 4),
- with a mixed-ability class of students with both males and females who will all sit either the Leaving Certificate Ordinary Level or Foundation Level paper,
- with a class of typical size (24 students),
- with the students' own teacher,
- within the students' own school,
- within the confines of normal school timetabled classes,
- with content that was curriculum aligned.

One could argue that using a Transition Year class meant the intervention was not normal as Transition Year is a year that is different from all the others in second level. It is a year that is set apart from the normal focus of the terminal exams of the Junior Certificate and Leaving Certificate. It is a year when students are more likely to be exposed to a greater range of pedagogical styles and there is also the opportunity to study content that is not limited to content from the Senior Cycle curriculum. Within the school where the research took place there is quite an emphasis, for the subject of mathematics, on studying content that is on the Senior Cycle curriculum. This lessens the criticism that using a Transition Year group means the intervention is not normal and strengthens the case that the learning experience could be replicated by others with other year groups.

A second argument to say the learning experience was not normal is that it was based in a computer room. The research is in the area of technology and therefore necessitated the use of computers. Using a set of tablet computers in the normal classroom for this group was considered but the internet speed was not sufficient for this to be practical. As the proliferation of access to more digital devices and greater internet speeds in classrooms (as opposed to some areas of a school) become more commonplace this type of learning experience could take place in a standard classroom and consequently make the learning experience more normal. As it stands, holding classes in the computer room is within the regular experience of students.

The above points mean the heuristics were robustly tested within the confines of what is a normal class setting. Arguably the greatest difference from Bray's successful interventions is the lessons were all 40 minutes or 80 minutes in duration, as this is the normal timetabled hours for mathematics lessons in the school. This had a great influence on the design of the learning experience. It was important that (a) the students would achieve some significant learning in the time and (b) the students would feel they had they had achieved something significant in the time, and as a consequence grow in confidence. Consideration was given to having activities which spanned a number of classes. The practicalities of students within Transition Year having many co-curricular and extra-curricular commitments influenced the choice to plan to have the tasks/problems to be completed within the confines of a 40 minute or 80 minute period. This meant the students were not going to be working on a small number of problems for a number of hours and this is a significant difference from Bray's interventions. It should be noted that Bray's heuristics do not specify that students should work on a problem for a long time but aspects of the heuristics, such as problem solving, investigation and using a model for 21st century learning and activity design, do lend themselves to longer tasks.

To ensure that (a) the students would achieve some significant learning in the time and (b) the students would feel they had they had achieved something significant in the time, and as a consequence grow in confidence the activities were designed to enable students to explore a problem or an aspect of functions through many representations and from an informal manner, such as a sketch or a typed prediction, to a more formal understanding of the problems. This scaffolding of the experience through a sequence of screens had to be balanced with giving students enough room to make and test conjectures, and construct their own mathematical knowledge, in what Freudenthal calls 'reinvention' of mathematical ideas (Little, 2009, p. 52). Hoyles (2016, p. 233) describes this delicate balance between scaffolding and freedom extremely well (emphasis in original):

> What we call *'landmark activities'* are designed so that students through their explorations with the software are bound to come up against inevitable epistemological obstacles. A major challenge – arguably, *the* major challenge – is then to design support for the student that provides enough freedom so they can actively engage in *their* task, yet with adequate constraints so as to be able to generate feedback that assists them to achieve *our* goals.

The activities chosen, modified and built from scratch for this learning experience were all designed with this delicate balance in mind. Some of the activities were selected from the "bundles" of activities on the website that are intended to be used sequentially. To tailor the activities to be in line with the research found in the review of the literature, the syllabus in Ireland and the prior knowledge and abilities of the participants it was not appropriate to choose many activities from any one bundle. Activities were chosen from the "Functions", "Linear" and "Modelling" bundles while other activities were modified or created by the researcher to ensure the activities were appropriate. Students came up against obstacles that needed to be overcome through exploration and discussion and each problem had to have enough freedom and constraints that the learners felt ownership of the activities and achieve the intended goal of the activities.

The next section outlines how the learning experience was designed in accordance with Bray's heuristics.

3.4.3 Bray's Heuristics in the Design of the Learning Experience

As the research question relates to applying Bray's heuristics this section takes each of the five heuristics, one at a time, and details the decisions made in the design of the learning experience that are a direct application of the heuristic.

3.4.3.1 Design Heuristic 1

The first design heuristic is that "activities should be team-based and encourage collaboration, in accordance with a socially constructivist approach to learning."

The participants were divided into teams of four. To implement a constructivist approach, create a need for discussion and encourage collaboration the problems for each of the activities were chosen because they were not straight-forward.

Small groups were chosen for a few reasons. Firstly, a meta-analysis on small groups compared with individuals learning with technology found that small group learning was more effective in terms of individual student achievement and also more effective for numerous affective outcomes (Lou, Abrami, & d'Apollonia, 2001). Secondly, groups are essential for implementing a constructivism type experience, which was found to be the most effective in Li and Ma's meta-analysis of computer technology on school students' mathematics learning (2010).

The participants were divided into teams of four based on including a more knowledgeable other in each team and also on which individuals would work well together. A reason for choosing teams of four instead of three was to ensure each team had a more knowledgeable other. The more knowledgeable others were identified by the researcher by looking at the students' results in previous exams, including the Junior Certificate, and also the work of the students in class. Identification of which individuals would work well together was based on previous interactions in class. The activities were designed to be quite difficult to tackle as individuals to encourage collaboration. With the help of more knowledgeable others within each team the activities were in the Zone of Proximal Development

(ZPD) for the individuals in each team (Vygotsky, 1978). This allowed each team to construct their own understanding of the mathematics i.e. reinvent the mathematics (Little, 2009).

It could also be argued that occasionally the software itself is acting as a more knowledgeable other. Activity Builder provides very useful feedback. For example, in the "Water Line" activity teams are asked to sketch the graph of water's height vs. time for various containers. The teams can check, through animation, how accurate their drawing of the height of the water against time compares with the actual water level i.e. each point on their graph is given meaning as the results are shown in the animation of the glass filling and is compared with the true level of the water. This is shown in the two water levels shown on the container in Figure 3.9 for a specific moment in time. The darker water level on the container represents the level of the graph the student has drawn and the white water level is the true water level. This clever and novel feedback acts like a more knowledgeable other by pointing out any slight inaccuracies and spurring the teams to improve.

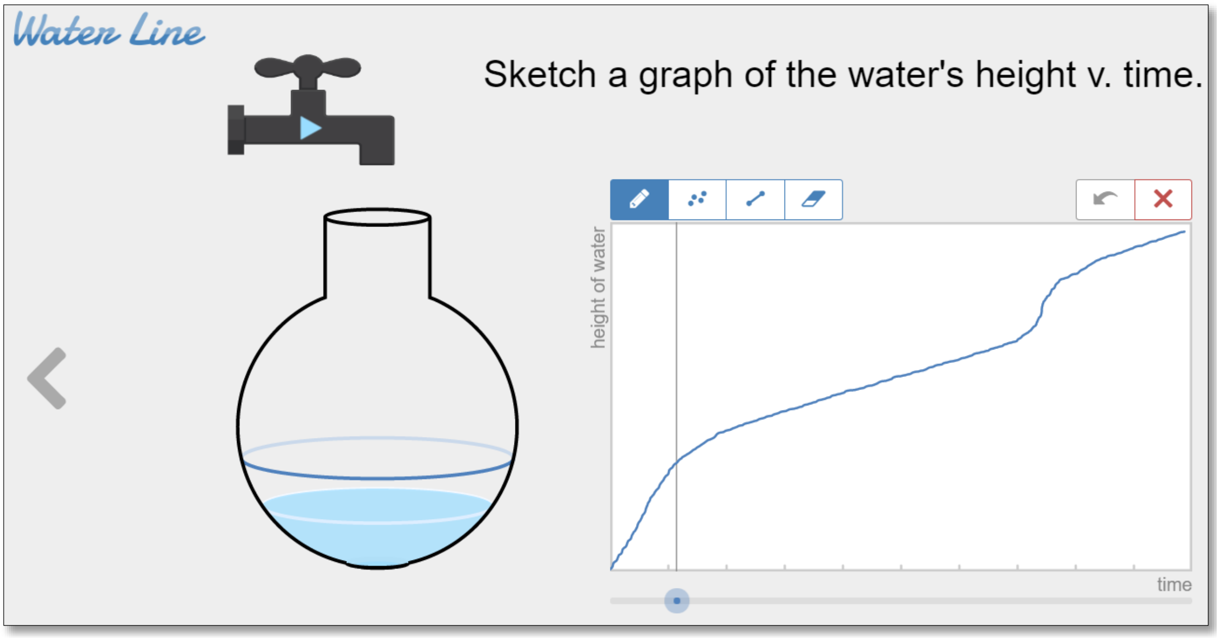

Figure 3.9 Feedback provided by Water Line

The activities include many opportunities for the teams to verbalise their understanding of a problem. This presents a challenge for the teams and encourages collaboration to verbalise their collective understanding as best as possible. This encourages collaboration and learning through interacting with one's peers.

3.4.3.2 Design Heuristic 2

The second design heuristic is that "activities should exploit the transformative as well as the computational capabilities of the technology."

The activities (i) chosen, (ii) modified and (iii) designed from scratch for the learning experience focus on some of the big ideas within functions e.g. covariation, modelling, analysing the effects of parameters and using multiple representations of functions (Carlson et al., 2002; Graham et al., 2010). Each of these big ideas could be learned using traditional tools but the chosen technology adds more to the experience. The technology provides students with useful feedback very quickly; in many cases much quicker than any teacher could possibly deliver. If this was the extent how the technology changed the activities it would place the activities at the Augmentation level of the SAMR model.

The homepage of the Desmos website says one of the three things the website offers is “Meaningful Feedback” and goes on to say “We show students what their answers mean, then give them the opportunity to improve their thinking and revise their work” (Desmos, 2017). The design of the activities anticipates students not getting the answer fully correct the first time and because it is so easy to have another attempt and get more feedback the students do have another attempt. For example, when students are sketching the graph of the water’s height vs. time for various containers in Water Line or when they are modifying the parameters of a function in Marbleslides it is unlikely they will get the correct answer immediately. The software gives them immediate feedback that what they have done is correct or incorrect and if it is incorrect, it shows in what way it is incorrect. The technology facilitates the students to make and test conjectures again and again. The technology facilitates this repeated process of making conjectures and testing them so well that it means that these activities would be very different without the technology and this is why much of this technology-enhanced learning experience could be considered to be at the Modification level of the SAMR model.

A second justification as to why much of the experience is at the modification level of the SAMR model is due to a feature called the “computation layer” (Desmos, 2016b). In one activity, called “Charge!”, the teams predict how long it will take a smartphone to reach full charge. The teams own predictions are referred to later in the sequence of screens. The teams create a line of best fit using limited data. Later, the teams own line of best fit is used to improve upon their first prediction and the students are asked to explain the meaning of both numbers in the equation of their own line of best fit. This feature uses data the students input to personalise later screens which separates the activities from what could be achieved from a static textbook or from following a single example a teacher could present on the board. This level of personalisation would not be possible without the technology and means that “Charge!” is an example of an activity in the transformative levels of the SAMR model. It has greatly modified the design of this type of an activity and places this learning activity at the Modification level of the SAMR model.

3.4.3.3 Design Heuristic 3

The third design heuristic is "activities should make use of a variety of technologies (digital and traditional) suited to the task, in particular, non-specialist technology such as mobile phones and digital cameras that students have to hand."

This heuristic is perhaps the one that features the least within the design of the learning experience. The traditional tools of pen, paper and calculators were provided to the teams. The intention of this was to facilitate students' explanations to one another. As well as this, having pen and paper to hand meant that if students decided to use their own methods to see if these matched the answers on the software this could also be done. It was foreseen that have these traditional tools would be especially useful for any activities that included formal algebraic skills.

3.4.3.4 Design Heuristic 4

The fourth design heuristic is "tasks should:

- involve problem-solving, investigation and sense-making,
- involve guided discovery,
- be situated in a meaningful/real context,
- move from concrete to abstract concepts,
- be open-ended but with constraints,
- be cross-curricular/cross-strand,
- be focused on skill development as well as on content,
- have a 'low-floor' and a 'high-ceiling'.

All of the activities fulfilled the first point as they involved problem-solving, investigation and sense-making. The activities encapsulated the third point by incorporating the ideas of RME by using problems that were chosen or designed to be engaging or experientially real for the students to create the opportunity for the mathematics to emerge. Examples of the starting point of these engaging contexts were predicting the price for a large number of pieces of Lego, predicting how long it would take a mobile phone to charge, an animation of a man being shot out of a cannon and predicting when two runners with very speeds would be at the same distance along a track based on limited information given in the form of a short video.

There are numerous examples of where the activities were designed to move students from concrete to more abstract concepts. In the Polygraph activity students are required to use more precise language to achieve better outcomes. Marbleslides is an activity that requires the students to develop a greater understanding of the three properties of the linear functions, namely (slope, y-intercept and domain). By being faced with ever more challenging problems a greater level of fluency with dealing with these parameters becomes necessary.

The cross-strand point is integrated into the design of the learning experience as while the activities are focussed on functions, the activities were also linked to algebraic understanding, algebraic skills and, in some cases, statistics. For example, in a couple of activities, the students created lines of best fit and used these lines of best fit to make predictions.

Developing skills in tandem with concepts is a point in Bray's heuristics and also an important point in the literature on approaches to developing students understanding in the area of functions (Friel & Markworth, 2009; NCCA, 2013b; Ontario, 2013) . The activities created by Desmos include such skills as sketching, plotting, forming functions and filling in tables. The activities created by the researcher include these skills and also include the algebraic and functional skills of substitution and solving equations.

The point about tasks having a low-floor and a high ceiling is incorporated into the design of the activities. Frequently teams are required to first use informal approaches to the problems e.g. a sketch or a prediction before more formal approaches, such as using substitution or solving an equation are called for to get a more precise answer.

3.4.3.5 Design Heuristic 5

The fifth, and final, design heuristic is "activities should be structured in accordance with the Bridge21 model (or a suitable structured alternative) of 21st Century Learning and activity design."

The Bridge21 Lesson Activity Template (Figure 3.10) was used, as much as possible, given the short duration of each class. If students had been working on the one problem for a number of hours it would have been easier to include all aspects of this template. Many of the components of the template were used in the learning experience and this is now described.

Figure 3.10 Bridge21 Lesson Activity Template

Each class began with a divergent thinking task such as to estimate the number of rugby balls that could fit into the sports hall of the school. Following the divergent thinking task the students began investigating a problem and discussing a problem using the technology. Teams prepared a PowerPoint presentation on what they had learned during the learning experience and presented this during the final lesson.

The previous five subsections demonstrated how the learning experience was designed in line with Bray's heuristics. The next section will provide an account of a pilot of some of the activities that were used for the learning experience.

3.4.4 Pilot of the Learning Experience

Some weeks before the learning experience a sample of the activities was trialled with a group of 2nd year students (two years younger than the research group). These students were an Honours Level group and the research participants were an Ordinary Level group. Two activities, designed by Desmos, were used with this group. The activities were Water Line and Desmos' Marbleslides. The pilot revealed that there was some novelty value attached with students going to the computer room for a class. The expectation of the students was that not as much work would be required. Overall the activities worked well as the students stayed on task for the duration of the lessons and they were challenged by the activities. There was evidence of learning as the students did not immediately solve any task. They had to work hard and collaborate to solve each task. The pilot revealed some small weaknesses within the activities e.g. asking students to solely explain their thinking on three screens in a row meant there was less enthusiasm for the third such screen. Finding weaknesses like this helped refine the content for the learning experience.

3.4.5 Length of the Learning Experience

The duration of the learning experience was 8 hours and 40 minutes. This is equivalent to three weeks of the participants' regular timetabled Transition Year mathematics classes. The rationale for the duration of the learning experience is based on research, practical considerations, having a learning experience which could be replicated by others and ethical considerations. These considerations are now detailed.

Li and Ma (2010, p. 233) established in their meta-analysis of the effects of computer technology on school students' mathematics learning that technology interventions that are six months or less are more effective than longer technology interventions. They suggest that this can be attributed to the novelty effects of technology decrease over time which "results in diminished motivation to use technology in a serious way for learning" (2010, p. 233).

Practicalities, such as acquiring ethical approval for conducting the research and a deadline which was less than 12 weeks from the time this approval was acquired meant the learning experience had to be shorter than 12 weeks. Other practicalities, such as holidays (3 weeks) and work experience (2 weeks) meant that the intervention would have to be less than 7 weeks in length.

To answer the research question the learning experience needed to be long enough to see some change in students' engagement and confidence. Bray's four in-school interventions varied from a very short intervention of two hours to three other interventions that lasted 10 to 12 hours each. It was seen as desirable to aim to have a learning experience that was of comparable length to the three longer interventions.

Another consideration for the length of the learning experience was the length of time it would take to teach a similar amount of content without technology. There is time in Transition Year to explore ideas in greater depth. For the research to be replicable by other teachers it would not be useful to take a considerably longer amount of time to teach the content using technology as it would to teach it without technology. Furthermore, it would not be ethical to spend an inordinate length of time on the topic, as the research is based on the work done as part of the participants' regular timetabled Transition Year mathematics classes.

This section described the rationale for the length of the learning experience. The next section will describe how each of the activities for the learning experience relate to Bray's heuristics.

3.5 Mapping the Activities to Bray's Heuristics

An in depth analysis was conducted of how each activity relates to each point of the heuristics. The results of this analysis are shown in Appendix 2. The analysis has an overview of the activity, the type of functional thinking the activity focuses on and the curriculum content the activity is focused on. This is then followed by a table which shows how the activities relate to Bray's heuristics.

3.6 Classification of this Research

Three recent research papers where technology was used to teach functions were classified in the previous chapter. This section summarises how this research is classified under each the four headings of Bray and Tangney's classification of mathematics education research.

Under the headings of 1. Technology 2. Learning Theory 3. SAMR Level and 4. Purpose, this research uses a *Toolkit* in a Social Constructivist way at the Modification level of the SAMR model for the purpose of a Change in Attitude (specifically engagement and confidence in the topic of functions).

Using a layout similar in style to the table used to classify 139 papers in the appendix of Bray and Tangney's paper (under review) this research would be classified in the following way:

Knox, T.	AN INVESTIGATION INTO BRAY'S HEURISTICS FOR MATHEMATICAL LEARNING ACTIVITIES AS APPLIED TO FUNCTIONS	2017	Tech: Toolkit LT: Social Constructivist SAMR: Modification Aim: Change in Attitude

3.7 Summary

This chapter outlined the design of this technology-enhanced learning experience. Countless decisions in the design of the learning experience were directed by the review of the literature in order to address the research question – can participation in a rich learning experience, designed in line with Bray's heuristics, improve student engagement and confidence when applied to the topic of functions within the confines of a normal school timetable?

The activities include multiple representations, modelling, adjusting parameters of functions and covariation as these are important aspects of functions. The choice of technology was based on whether it could facilitate exploration of these aspects of functions as well as being able to collect data about students' work, ease of use for students, and being able to facilitate the creation of activities that engage students in the topic of functions. The design of the experience draws from RME to ensure the activities are engaging in order to fulfil one of Bray's heuristics. Other aspects of the design of the learning experience, and how these aspects relate to the heuristics were also presented. The design includes dividing the participants into teams of four to be faced with challenging activities to encourage the students to collaborate and construct their own knowledge together. To integrate the Bridge21 model into the learning experience the Bridge21 Lesson Activity Template was used, as much as possible, given the short duration of each class. A high level of technology integration was also used, with many activities being at the Modification level of the SAMR model.

This chapter described how the learning experience was informed by the literature, especially Bray's heuristics, as these heuristics are central to the research question. The next chapter describes the implementation of this design, the types of data collected and how this data was collected in order to answer the research question.

4 Methodology

4.1 Introduction

This chapter begins by outlining the research question that was investigated by this research project. The implementation of the design of the learning experience is described. Following this, the chosen research methodology is described as well as the rationale for its choice. The research methods are outlined and the rationale for choosing them is explained. The data collection, data preparation techniques and ethical considerations are outlined and the chapter ends with an analysis of the limitations of the approach chosen to investigate the research question.

4.2 Research Question

The research question is can participation in a rich learning experience, designed in line with Bray's heuristics, improve student engagement and confidence when applied to the topic of functions within the confines of a normal school timetable?

4.3 Implementation

The participants were recruited from the researcher's established Transition Year mathematics class which is in a mixed post-primary school. The learning experience was part of the participants' regular timetabled mathematics classes. The class was a typical class size. The 24 participants were of mixed gender and were 15-17 years of age. In Transition Year in this school there are two mixed-ability classes for students who will sit either the Leaving Certificate Ordinary Level or Foundation Level paper. These two class groups would represent 40% of students within the year group. The other 60% of the year group intend to sit the Leaving Certificate Higher Level paper. The duration of the learning experience was thirteen 40 minute classes, which are a combination of 40 minute and 80 minute periods. The learning experience took place in the school's computer room.

4.4 Research Methodology

The research methodology used was an exploratory case study which is appropriate as it facilitates investigating a contemporary phenomenon in depth and within its real world context (Yin, 2013, p. 16). A classroom has many important contextual variables that could affect any learning experience. In order to understand the learning experience and to observe if there is any change in students' level of engagement it is important to capture as much of the experience in as deep a way as possible. A case study is suitable for this.

There are several types of case studies. Cohen *et. al.* (2007, pp. 254-255) describe how they can be categorised in a multitude of ways. One such method is to do so based on the outcomes. A descriptive case study provides narrative accounts of what happened. An exploratory case study can be used as a pilot to other studies and an explanatory case study can be used to test theories. The type of case study chosen for this research is an exploratory case study. This recognises the limitations of the research as it is unlikely that the findings from such a small sample would be suitable for explaining the causes for any findings. The evidence might suggest or indicate certain reasons why things happened but they would not be enough to fully explain the reasons for the findings. Yin (2013, p. 17) says case studies rely on many sources of evidence and these sources of evidence are described in the next section.

4.5 Research Methods

A parallel mixed methods design (Creswell, 2008, p. 557) was used for this exploratory case study. Mixed methods were chosen to ensure that the richness of the learning experience was captured. Mixed methods exploit the advantages of both quantitative and qualitative research and reduce the limitations of using just one of either quantitative or qualitative (Creswell, 2013, p. 218). The Mathematics and Technology Attitudes Scale (MTAS) can identify changes in student's attitudes in certain areas but it cannot provide information as to why those changes may have occurred. The depth and variety captured through researcher observation, through analysing the data captured by the website and through group interviews supplemented the data collected through the MTAS and provided opportunities for triangulation to gain a greater insight into the research question.

4.5.1 Quantitative Data

Quantitative data was collected through administering the MTAS (Pierce et al., 2007) prior to the learning experience and after the learning experience. It was chosen for three reasons. The first reason was its suitability for the focus of the research, which is how a technology-enhanced learning experience can affect student engagement and confidence. The MTAS is designed to measure changes in students' attitudes to mathematics and technology. The second reason is that it is a validated data collection instrument (Pierce et al., 2007). The third reason is that this research is building on Bray's research and so it appeared sensible to use the same tool.

There are five subscales in the MTAS: behavioural engagement (BE), confidence with technology (TC), mathematical confidence (MC), affective engagement (AE), and attitude to using technology for learning mathematics (MT) (Pierce et al., 2007). Confidence with technology, mathematical confidence, and attitude to using technology for learning mathematics are all self-explanatory. Affective engagement is concerned with how a student feels about mathematics while behavioural engagement is concerned with how students behave when learning mathematics.

The MTAS is a 20 question survey instrument. Every group of four questions relate to each of the five subscales. A copy of the MTAS is shown in Appendix 5. The MTAS contains statements, such as "Learning mathematics is enjoyable" and five boxes for the respondent to choose from to indicate their level of agreement with the statement. The choices are strongly disagree, disagree, not sure (if they agree or disagree), agree and strongly agree. After the respondent answers all 20 questions the responses of strongly disagree to strongly agree are matched to the numbers 1 to 5. The totals for each subscale is found by adding the numbers for the four questions in that subscale. Each subscale has a minimum score of 4 and a maximum score of 20.

Before the participants completed the MTAS for the first time the participants were given unique 3 letter codes e.g. DYH, and were asked to keep these codes in a safe place (or safe places). This was to ensure the participants could retain their anonymity and also so that pre- and post- data could be paired.

4.5.2 Qualitative Data

Qualitative data was collected in a number of ways. During the learning experience students had access to pen and paper. This work was collected by the researcher. Each screen that is completed in the activities is automatically saved. This provided extensive data about the engagement of students with the activities. The data gained from the software was examined after each lesson. The researcher took field notes on each class period.

The researcher's observations yielded some qualitative data. Feelings, perceptions, intentions, changes in attitude are all relevant to engagement and confidence and cannot be observed. To investigate these unobservable elements three group interviews were conducted after the learning experience was finished. The researcher is aware that there are disadvantages with using a group interview. A one-to-one interview would have had a greater chance of collecting richer and deeper data and there would have been no possible complications regarding group dynamics. The comfort of the participants, from an ethical point of view is paramount and group interviews are less intimidating for young students (L Cohen, Manion, & Morrison, 2011, p. 433) so group interviews were chosen.

Random samples of size 4 were selected from all the participants for the group interviews. The intention was that there would be 4 participants in each group but one participant decided they did not want to participate just before one interview began. Each group interview consisted of either 3 or 4 participants. Each of the 3 group interviews lasted approximately 20 minutes.

The type of interview chosen was a semi-structured one. The interview was semi-structured so that if anything arose in the interview it could be explored further. The protocol for these group interviews and a list of the planned questions is provided in Appendix 6. The questions were chosen to acquire in depth qualitative data on many aspects of the learning experience. Areas, such as engagement and confidence, the design of the learning experience (including aspects of the design heuristics) and participants' opinions on working in groups, and using technology, to learn mathematics were included in these questions in order to answer the research question.

Some of the questions were open and related to the experience as a whole. Some questions were based on aspects of the design of the learning experience (including aspects of the design heuristics). Other questions were chosen to aid triangulation with the quantitative data from the MTAS. Some of these questions were planned prior to the learning experience but there was an emic element to how the questions developed as the interviews progressed. For example, one of the first two groups identified an aspect of the technology that they found useful and this was asked about in the final interview.

4.6 Data Preparation

The pre- and post- learning experience data from the MTAS was inputted by the researcher into a Microsoft Excel spreadsheet. The totals for each of the subscales for each participant were calculated. The mean scores of the participants before and after the learning experience could then be calculated. The Analysis ToolPak Add-In in Microsoft Excel was added to Excel in preparation for doing a paired two sample *t*–test with the totals for each participant for each subscale.

The audio recordings of the three group interviews were listened to in full. These interviews were then transcribed by the researcher using Microsoft Word. Details on how the transcripts were coded are given in the next chapter.

4.7 Ethical Considerations

As the research involved human participation ethical approval to conduct the research was sought and received from the Research Ethics Committee of the School of Computer Science and Statistics at Trinity College, Dublin. The email of this approval is in Appendix 3.

A detailed information sheet was given to the Board of Management so they could give informed consent. Consent was sought and received from the Board of Management of the school. The Board of Management information sheet and consent form are in Appendix 4.

The participants were informed about the nature of the research and were reminded that their participation was fully optional and that the purpose of the study is to test educational theory and resources. As the research involved the participation of minors consent of parents/guardians and assent of participants was sought and received before the research was undertaken. A detailed information sheet was given to parents/guardians and to the participants so they could give informed consent/assent. The information sheets and consent forms for Parents/Guardians and information sheets and assent forms for participants are shown in Appendix 4.

A conflict of interest was that the participants were students in a class the researcher teaches. Students were reminded that their participation in the research was fully optional and that the purpose of the study is to test educational theory and resources and not to test them. Participants were reminded that they were free to withdraw from the research at any time.

Identification of individuals was another consideration to be mitigated against. This possible identification could have occurred during open responses to the group interview questions. Participants were advised not to refer to any other participant by name or in any way that may identify themselves or another person. As part of the protocol for the group interviews the participants were told "Please do not name third parties in any answers. Any such replies will be anonymised." The interviews were audio recorded. The audio recordings were stored digitally on the researcher's password protected PC until the research project was submitted and they were then deleted. Only the researcher had access to the recordings until the recordings were destroyed.

The learning experience centred upon the use of Desmos Activity Builder, a third party website. The participants did not sign up to the website so the website did not receive any personal information about the participants. The participants worked in teams. The website requires a name to begin each activity and the participants submitted "Team 1", "Team 2" etc. This meant that no individual participant's personal data was stored by the Desmos website.

4.8 Limitations

This study looked at one class group for a short intervention of approximately 8 hours. It is not possible to make any generalisations to the population as a whole from the findings from this one group. It may, however, be acceptable to make some tentative generalisations to groups that are similar to the one studied. Given that there are many classes with similar mathematical abilities being taught in classes with a similar number of students, within the confines of 40 and 80 minute classes for similar content means the findings from this research could possibly be generalised to many similar classrooms across Ireland. With one computer shared between a team of four and using freely available web based tools it means similar technology-enhanced learning experiences can be replicated across Ireland. Given that the content is not specific to Ireland similarly designed experiences could be used in other countries.

The group interviews were conducted by the researcher, which may have led to bias in the answers given by the participants. To mitigate against any bias caused by the researcher conducting the group interviews the following was stated at the beginning of the each group interview "I want you to be totally honest when answering the questions and not to say what you think I might want to hear, but say what you really think. There are no right or wrong answers. This interview is about your opinion." This was not ideal, but was adequate for the purposes of acquiring rich data pertinent to answering the research question. The alternative of having someone not familiar with the learning experience conducting the interview was considered, but this would have meant this individual would have been less likely to probe further into anything relevant that turned up in the course of an interview.

Another limitation was the researcher is not an experienced interviewer and this had the potential for opportunities for further exploration of points raised to be missed.

Some of the qualitative data collected throughout the learning experience was the field notes made by the researcher. The research would have benefitted from an additional observer collecting data about the learning experience in a more systematic way.

4.9 Summary

This chapter outlined the research methodology and research methods chosen to investigate the research question and detailed why they were appropriate. The implementation of the design of the learning experience was described. The data collection methods and the reasons for choosing them – including aiding triangulation of data - were described. The data preparation techniques and ethical considerations were also outlined. The final section of the chapter commented on the limitations of the approach chosen, and how this approach was implemented, to investigate the research question.

The next chapter will report how the data was analysed and report on what the research revealed.

5 Findings and Analysis

5.1 Introduction

This chapter presents an analysis of the data collected through use of the Mathematics and Technology Attitudes Scale (MTAS), researcher observation, data captured by the website, and a number of group interviews. Firstly the analysis of the MTAS data is presented. This is followed by the results from the three group interviews and is supplemented by some data collected via researcher observation and from the website.

5.2 MTAS Results and Analysis

5.2.1 Analysis using Categories of Levels of Positivity in Attitude

As stated earlier, each subscale in the MTAS has a minimum score of 4 and a maximum score of 20. Barkatsas (2012, p. 158) states the scores that match very positive, moderate and either neutral or negative attitudes. 17 or above are considered to indicate very positive attitudes, 13-16 is considered moderately high and 12 or less indicates a neutral or negative attitude to the domain being measured.

The COUNTIF function on Excel was used to count the number of students within each category for each of the subscales for the pre- and post- learning experience data to see what changes, if any, there were in these broad categories. The pre-learning experience percentages within each category for each subscale of the MTAS are shown in Table 5.1.

	Neutral or Negative	Moderate	Very Positive
BE	29%	67%	4%
TC	17%	54%	29%
MC	63%	33%	4%
AE	63%	38%	0%
MT	46%	42%	13%

Table 5.1 Pre-Learning Experience Percentages Within Each Category

The pre- and post- data was analysed to see if there were many students that moved from one category to another. There was little change in the numbers in each category for BE, Figure 5.1, a small increase in the numbers with a more positive attitude in TC, Figure 5.2, and little change in the numbers in each category for MC, Figure 5.3.

There were large increases in the numbers of students with more positive attitudes in AE, Figure 5.4, and MT, Figure 5.5. For AE the numbers in the Neutral or Negative category reduced and the numbers in the Moderate category increased. There was no change in the number in the Very Positive category; this remained at zero. For MT the numbers in the Neutral or Negative category reduced and the numbers in the Moderate and Very Positive categories increased.

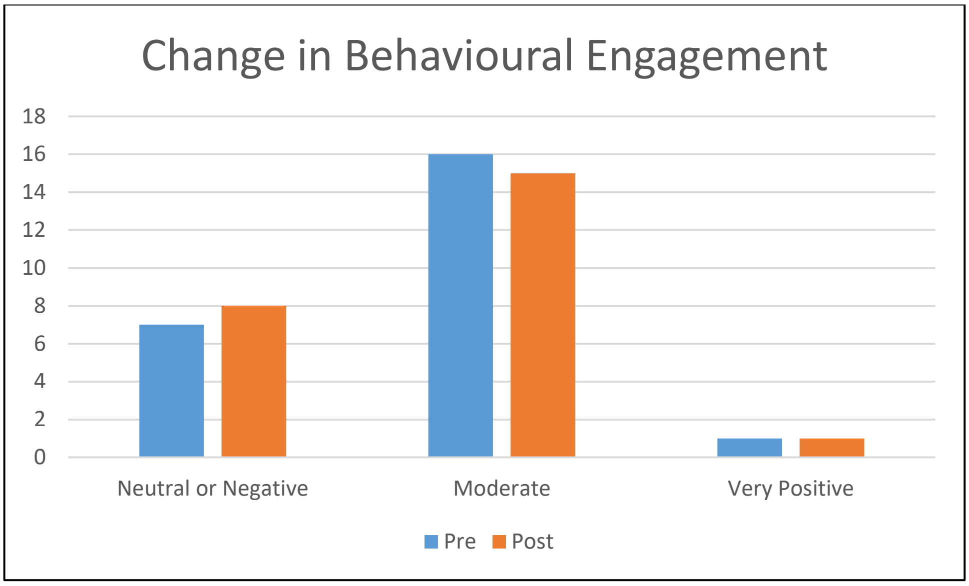

Figure 5.1 Change in Behavioural Engagement (BE)

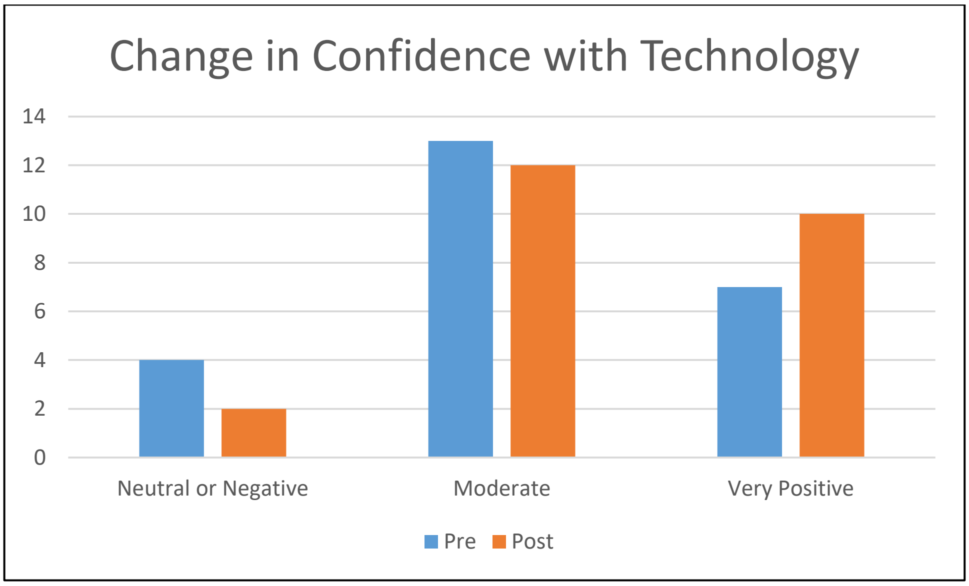

Figure 5.2 Change in Confidence with Technology (TC)

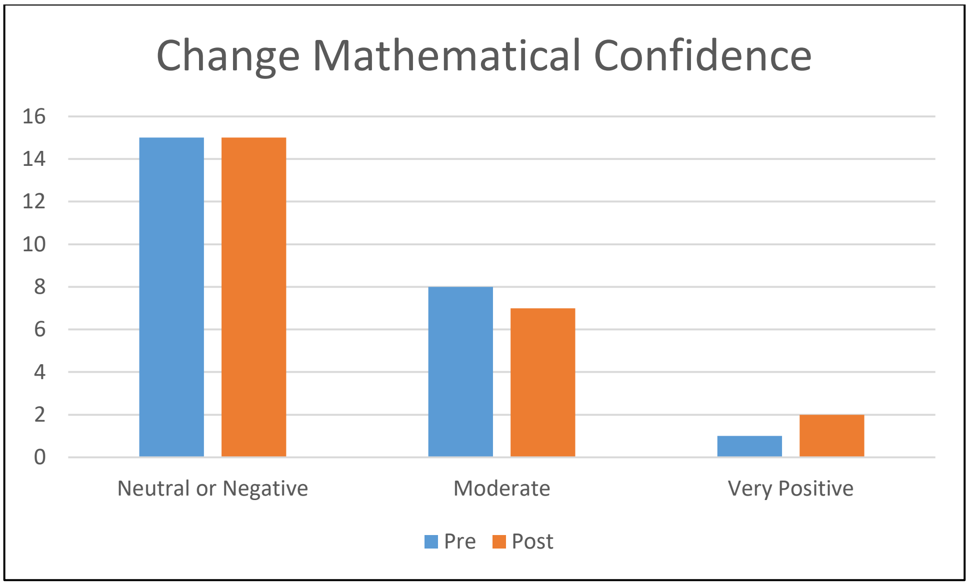

Figure 5.3 Change in Mathematical Confidence (MC)

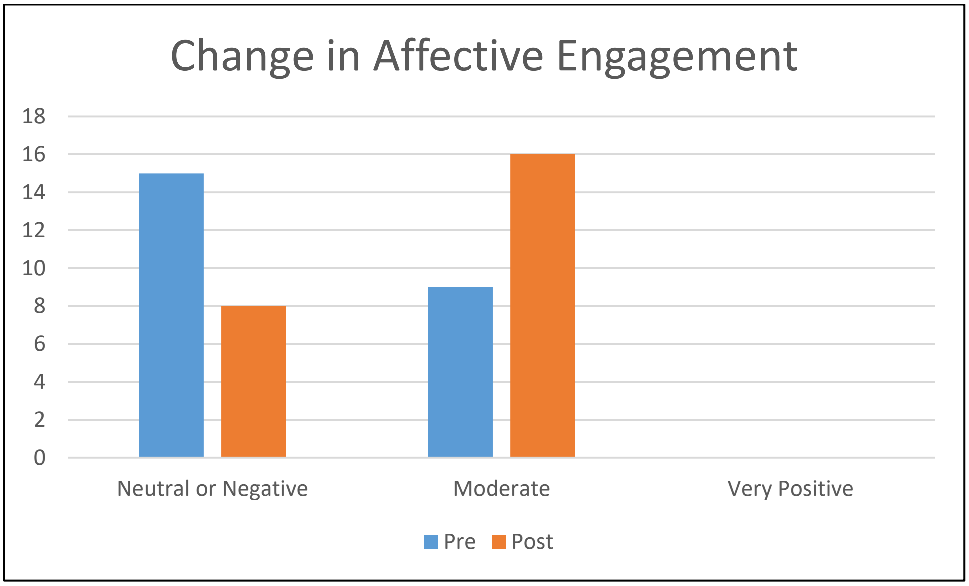

Figure 5.4 Change in Affective Engagement (AE)

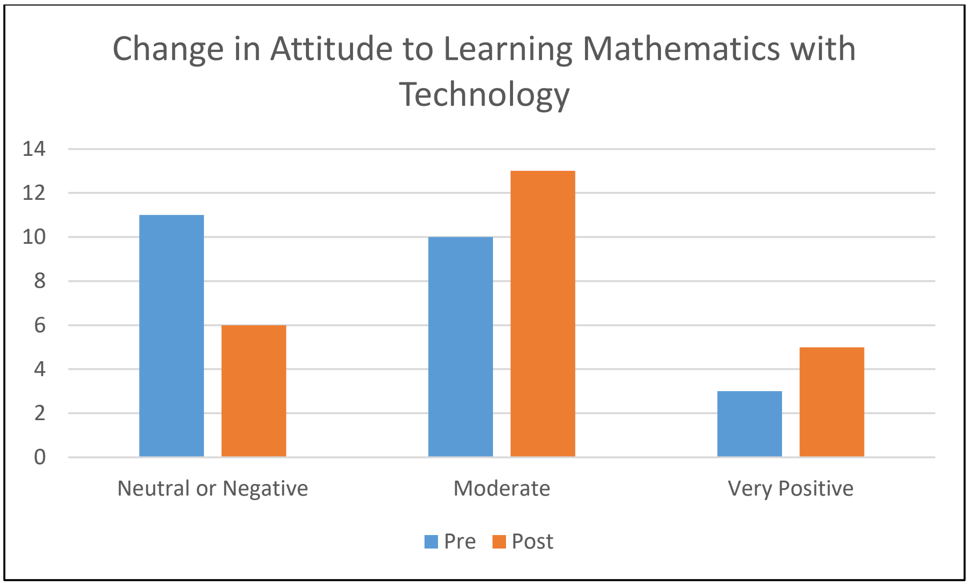

Figure 5.5 Change in Attitude to Learning Mathematics with Technology (MT)

5.2.2 Analysis Using a Comparison of Mean Scores

The mean score for each subscale for post- learning experience data was then calculated and compared to the pre-learning experience data. These means are shown in Table 5.2. There were increases in the mean scores for all five subscales, Figure 5.6.

	Mean-pre	Mean-post
BE	13.5	13.6
TC	14.3	15.8
MC	10.6	11.5
AE	11.5	12.8
MT	12.7	14.6

Table 5.2 Pre- and Post- Learning Experience Means for the MTAS Subscales

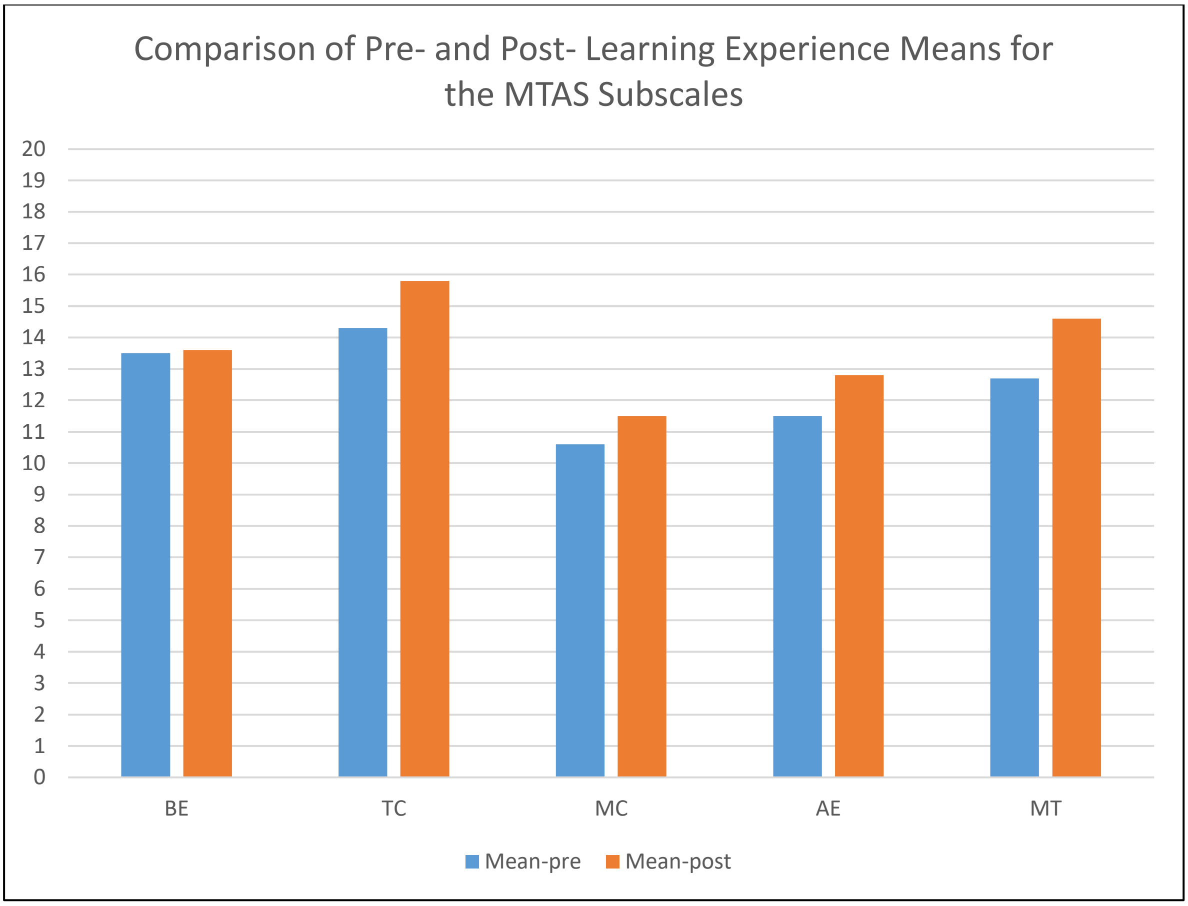

Figure 5.6 Pre- and Post- Learning Experience Means for the MTAS Subscales

A more in depth analysis of the data was conducted to see if any of the changes in mean scores were significant. With a sample size of 24 (n = 24) the number of degrees of freedom for a t-test is 23 (n–1). For the purposes of this research the researcher chose a level of significance of 0.05, α =0.05. This is a commonly used threshold value for hypothesis testing (Louis Cohen et al., 2007, p. 519; De Veaux, Velleman, & Bock, 2008, p. 508) and is adequate for this research.

To determine if there was a difference between the pre- and post- learning experience data a paired two sample t–test for means was conducted using the Analysis ToolPak Add-In in Microsoft Excel. The researcher consulted the Students t-distribution table (De Veaux et al., 2008, pp. A-62) and determined that for a two-tailed t–test, with 23 degrees of freedom and an alpha level of 0.05, the critical values are –2.069 and 2.069. This means that a t-statistic of less than –2.069 or greater than 2.069 is significant. The differences in mean scores for BE (0.267), TC (1.934) and MT (1.131) were not significant. Significant differences were found for two of the subscales; AE (2.734) and MT (2.187). Table 5.3 shows the means and standard deviations for the pre- and post- learning experience data as well as the t–statistic for each subscale.

	Mean-pre	SD-pre	Mean-post	SD-post	$t(23)$	p
BE	13.5	2.0	13.6	2.2	0.267	0.792
TC	14.3	3.4	15.8	2.3	1.934	0.065
MC	10.6	3.7	11.5	3.5	1.131	0.270
AE	**11.5**	**2.4**	**12.8**	**2.6**	**2.734**	**0.012**
MT	**12.7**	**4.0**	**14.6**	**3.9**	**2.187**	**0.039**

Table 5.3 Results of Two Sample t–tests for Means

The Analysis ToolPak was also used to calculate the *p*-values for each sample mean. With $\alpha = 0.05$ a *p*-value of 0.05 was used. Table 5.3 shows the *p*–value for each subscale. For AE, as $p = 0.012$ is less than 0.05 this result is significant. With a *p*–value of 0.012, if there was no difference in the mean values for AE, a difference this large, or larger, would occur only 12 times in 1000. For MT, as $p = 0.039$ is less than 0.05 this result is significant. With a *p*–value of 0.039 if there was no difference in the mean values for MT, a difference this large, or larger, would occur only 39 times in 1000. The *p*-values for all subscales is shown in Table 5.3.

The focus of this study was to improve students' engagement and confidence in mathematics, specifically in the area of functions. The learning experience was somewhat successful in the area of engagement. The greatest change occurred in the mean scores for Affective Engagement, AE, and this was statistically significant. This points to the fact that the design of the learning experience influenced this change. However the tiny changes, although positive, in Behavioural Engagement, BE, and the small change in Mathematical Confidence, MC, were not what was envisaged when the learning experience was being designed.

The large change in students' Attitudes to Learning Mathematics with Technology, MT, is in line with the expectations from the design of the learning experience. This could possibly be something that could be capitalised upon to help this set of students and similar student cohorts to learn maths. There is also, possibly, a link here to the change in Confidence with Technology, TC.

5.3 Group Interview Data

The typed transcripts were printed and read through fully first to conduct an exploratory analysis to become sensitised to the data (T. Savage, personal communication, December 2, 2016). The data was then hand coded. Firstly, open coding was done to see what would emerge without preconceptions. Segments of text were highlighted and labelled with a short text code. For example "Tech. Pos." was used if the segment of text indicated technology had some benefit. A sample of the open coding is shown in Appendix 7.

The codes were inserted into a table in Microsoft Word. As each code was added a short description of each code was added in the next column and the number of each code found was increased in another column. As themes emerged similar codes were put near one another in the table to make it easier to count any new occurrences of the code. 77 different codes were generated. 485 segments of text were coded. When all the codes were added to the table the codes were categorised more formally into themes

The themes that emerged concerned “Normal Class”, “Attitudes”, “The Experience as a Whole”, “Engagement”, “Learning”, “Groups”, “Technology”, “Presentations” and the “Format” of the experience. This is shown in Appendix 8. There were specific questions relating to all of these themes, so it is not surprising that these themes emerged, but it is also worth noting that many of them are interlinked e.g. a question about technology yielded opinions on technology, learning, groups and engagement.

After this the table was reviewed to see if there were any findings that could be drawn from the table. This was first done before concentrating on the research question (to look at the data in an open way initially). It was planned to do both open coding and then do directed coding from the point of view of the research question, the design heuristics and the MTAS subscales. Even a cursory review of the themes that emerged from the open coding would show that the themes related strongly to the themes of this research, so directed coding was deemed unnecessary.

The research question was reviewed and the data was explored again from this perspective. Following this, the design heuristics were examined and the data was investigated from this angle. Finally, the MTAS subscales were inspected and the data was scrutinised again from this viewpoint.

The ideas that appeared most often were related to learning within a team, the benefits of technology, that mathematics was learned, the experience was positive, the experience had some challenges within it and that challenges were overcome within the experience. These relate to a learning experience that was designed to have enough challenge within the engaging activities that it encouraged students to learn mathematics from each other with the aid of technology. The plan was that by overcoming the challenges in the activities the students would increase their levels of engagement and confidence in the subject.

The findings that were revealed in relation to the group interview data will be outlined in the next section. They are organised into sections relating to the five subscales of the MTAS, the design of the learning experience and the Bridge21 model.

5.4 Findings

This section will present the findings from the analysis of the data described in the previous sections. This section will present any congruence in the data sets to support any findings. Any incongruities between the sets will also be reported.

5.4.1 Behavioural Engagement

The quantitative data showed only a marginal increase in the mean scores for BE from 13.5 to 13.6 and there was little change in the numbers in each of the categories Neutral or Negative, Moderate and Very Positive.

The statements used to measure BE in the MTAS (Appendix 5) concern concentration, willingness to answer questions asked by the teacher and perseverance. Researcher observation throughout the experience found that there was a greater level of focus and perseverance from the majority of students for the majority of the time in comparison to the normal behaviour from the participants.

The activities for the learning experience were chosen to be at a high level to encourage team collaboration. There was a lot of evidence from the group interviews that students needed some perseverance to complete the activities as students found the activities challenging (*f*=31) but were able to manage them (*f*=24). This tallies with the observations made during the learning experience. There were frequently times when groups got stuck and had to try again and again to achieve a satisfactory solution.

"It [the learning experience] was just kind of more interesting, like it made it easier and more interesting when you're trying to figure it out."

> *"Some parts were kind of challenging and difficult but the others were easy."*
>
> *"There was a few there that were hard but we always got them in the end."*

Marbleslides, had many levels that students needed to discuss with one another in order to be successful and many attempts were needed to overcome a challenge such as the one shown in Figure 5.7.

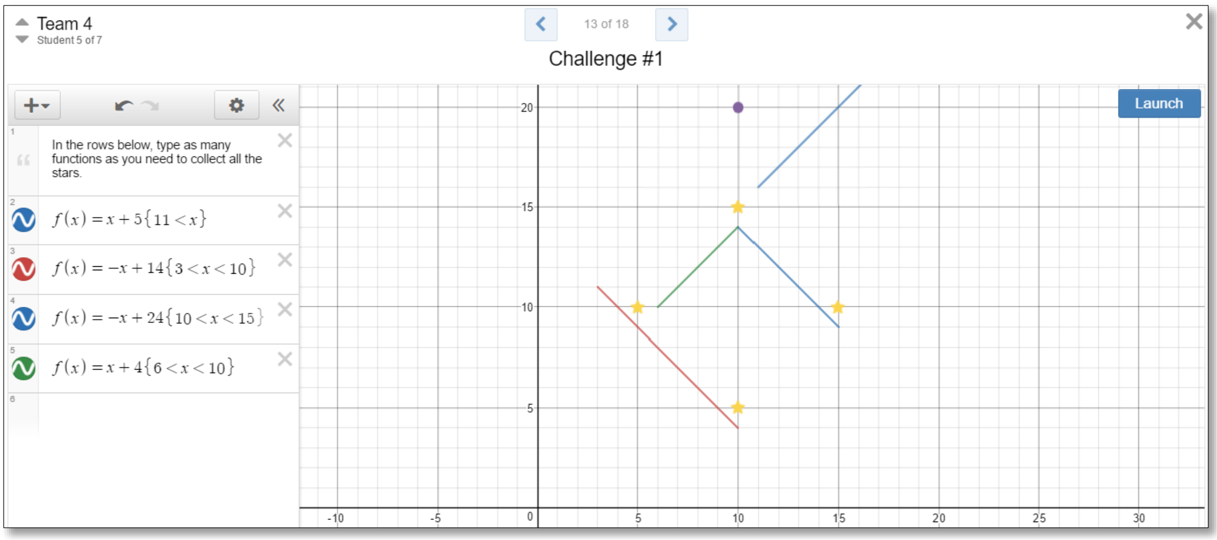

Figure 5.7 A Marbleslides Challenge

The finding is that the behaviour communicated in the group interviews and the change in behaviour observed by the researcher has not transferred to a change in attitude in Behavioural Engagement.

5.4.2 Confidence with Technology

The quantitative data for Confidence with Technology showed a decrease in the numbers in the Neutral or Negative and Moderate categories and an increase in the numbers in the Very Positive category. The mean scores for TC increased from 14.3 to 15.8 which is indicative of some change in attitude. The *p*-value of 0.065 shows that there was no significant improvement in this subscale.

This was the subscale with the highest mean before the learning experience. This is not surprising given how pervasive technology use is among this age group. A criterion for choosing the technology was that it has low-floor to be able to use it. The students had very little problems with navigating and interacting within the technology during the learning experience and it was rare if help was required from the researcher during the experience. Occasionally the software froze or the connection to the website would be lost. The participants were able to separate their own abilities in using the technology from any technical issues that were the fault of the website, the computers or the internet connection.

The finding is that there were some marginal gains in Confidence with Technology in this subscale as the participants' confidence grew slightly from a high base as they used the relatively straight-forward technology.

5.4.3 Mathematical Confidence

Mathematical Confidence was the subscale with, by far the lowest mean, 10.6, before the learning experience. The quantitative data for MC showed no change in the numbers in the Neutral or Negative category and only a decrease of one in the Moderate category and an increase of one in the numbers in the Very Positive category. The mean scores for MC increased from 10.6 to 11.5 but the *p*-value of 0.270 shows that there was no significant improvement in this subscale.

Given the frequency of times learning maths (23) and was referred to in the group interviews it makes sense that the marginal change in this subscale was in the positive direction. Other data from the group interviews concurs that there was little change in this subscale. For two of the group interviews, when asked if the learning experience had changed their confidence in using and understanding maths the most common responses were "not really" or "no". In the other group interview the responses were:

> *"Ah, a little bit yeah. I feel more confident doing functions. I didn't understand them at all before, but now I have some understanding"*
>
> *"Yeah I feel I can speak out more, I wouldn't keep it to myself."*
>
> *"I think it's gone up, yeah."*

The researcher observed many moments where the students experienced success during the learning experience, but the small change in the mean score in subscale was not statistically significant.

5.4.4 Affective Engagement

There was a statistically significant difference between the pre- and post- learning experience mean scores for Affective Engagement. The mean score changed from 11.5 to 12.8. The *p*-value was 0.012. The majority of the participants had Neutral or Negative scores for AE prior to the learning experience. The numbers in the Neutral or Negative category reduced and the numbers in the Moderate category increased. There was no change in the number in the Very Positive category; this remained at zero.

The statements used to measure AE in the MTAS (Appendix 5) concern interest in learning new things in mathematics, mathematics being enjoyable, whether one gets a sense of satisfaction from solving mathematics problems and whether in mathematics you get rewards for effort.

The first question in the group interviews asked the participants to describe their normal maths class. In answering other questions references were also made to the students' normal maths class. From the coding of the transcripts the most common things said about the normal maths class were that they felt positive about the class (*f*=10), maths is learned in the class (*f*=9) and learning in the normal class benefits from the group work that occurs (*f*=7). The other references to the normal class were that the class can be boring or uninteresting (*f*=5), the class is normal (*f*=4), the class is enjoyable (*f*=1), there were neutral feelings about the class (*f*=1) and one interviewee said the teacher works or says a lot (*f*=2). While there are some positives in this data about the normal class there is probably a low expectation attached to saying a class is normal (*f*=4), quite a few mentions of the class being boring or uninteresting (*f*=5) and quite a low number saying the class is enjoyable (*f*=1) (there was no specific question on whether the class was enjoyable or not).

In contrast, the learning experience was seen as positive (*f*=32), engaging (*f*=29) and enjoyable (*f*=8). One comment said the experience was negative (*f*=1) (expressing a preference for the normal class) and there was another comment with a neutral view on the learning experience (*f*=1), "it was alright". Some of the reasons for the high frequency of positive comments may have been to do with the fact that it was quite different to the normal class (*f*=21). There seemed to be some level of novelty value with the learning experience as it was said that they felt more positive about experience at the start (*f*=6). However, the participants also said they found aspects of the experience boring (*f*=9) and repetitive or in need of more variation (*f*=14) as the learning experience progressed, while it was also said the activities were varied (*f*=2).

> *"I thought they were beneficial, they were like really good and kind made you think about math in different ways"*

"It was more interesting and I found that I looked forward more to coming to maths"

"The problems were asked in kind of a more modern way, so...it was kinda easier to relate"

"TK: Did you find the tasks interesting?

3: Yeah, I thought they were. I thought they were, like, well laid out for our age group and they just sort of kept you interested. They weren't sort of just looking at lines the whole time, like I said, you were doing Lego you know the running and shooting the person in the air with the parachute and all that."

Researcher observation would concur with the fact that many of the students found the activities engaging for the majority of the time; especially in comparison to the normal level of engagement of the class. The researcher attributes this to choosing, modifying and making activities that captured the imagination of students and allowed them to discover and reinvent some mathematical ideas themselves. Some teams worked with a level of urgency that was not seen by the researcher for the same individuals in the normal class setting. As mentioned earlier there was a greater level of perseverance in the students to complete the activities and there was visible enjoyment from working together to solve problems and satisfaction, and even celebrations, when certain problems were solved.

The MTAS statements for AE concern interest, enjoyment, satisfaction and reward. The finding is that the majority of the students did find the learning experience a positive one that was engaging and enjoyable based on a statistically significant change in mean scores, researcher observation and the majority of the group interview data. There were some views expressed that the experience needed more variety towards the end and this will be discussed in Section 5.4.7.

5.4.5 Attitude to Learning Mathematics with Technology

There was a statistically significant difference between the pre- and post- learning experience mean scores for Attitude to Learning Mathematics with Technology. The mean score changed from 12.7 to 14.6. The p-value was 0.039. The numbers in the Neutral or Negative category reduced and the numbers in the Moderate and Very Positive categories increased.

The statements used to measure MT in the MTAS (Appendix 5) concern preferring to use technology for learning mathematics, technology being worth the extra effort in mathematics, mathematics being more interesting when using technology and technology helping the learner learn more effectively.

Technology was referred to throughout the group interviews and was frequently mentioned in a way that referred to it being beneficial to the learning experience (f=32) for reasons, such as increasing engagement, being different to the norm, making it easier to attempt a question again, being able to manipulate something and see the effect of this on the screen (f=2) and being better for those with different learner needs (*f*=6). There were some references to using technology being a disadvantage (*f*=5). For example, it was expressed that it is awkward if one needs to move between the technology and paper (*f*=1) and technical issues (*f*=3), such as the computer freezing or having to redo work if the connection was lost were also unhelpful.

> *"Just doing the graphs when you're filling [in] like functions and stuff like you kind of when you saw where it kept moving to you understood more the y-axis and the x, where everything went, like"*
>
> *"4: Em, Kind of like helpful because you could go back if you made a mistake, you can go back and cross out all the things like you might do when working in a classroom and start over again.*
>
> *3: Yeah, I would have said the same, because it might be all messy and like crossing stuff out so like you could just like take it all back and then write it all so it looked like better. Em, I think, negative kind of like .. sometimes it kept clicking out of it and it was really annoying because you were in the middle of doing something but other than that I thought it was like actually really helpful.*
>
> *2: I thought it was helpful because you got to see like every time you changed a number and like something would happen on the screen and you'd see something and that helped."*
>
> *"Helpful because you had to work a few times at finding the numbers you needed for the graph."*

The greatest advantages observed by the researcher was how the technology facilitated the teams to have another go at solving a problem. In many cases this was tied in with the instant feedback the technology gave that the answer needed to be changed or improved upon. In each lesson the students gained from the instant feedback the technology gave the students. Figure 5.8 shows the Teacher Dashboard for an activity where the students took many attempts at improving their answers as the technology made it easy to do so. During the pilot and the study itself even the most minor differences from the fully correct answer in some of the activities resulted in plenty of discussion and in most cases the teams trying to draw an even more accurate graph.

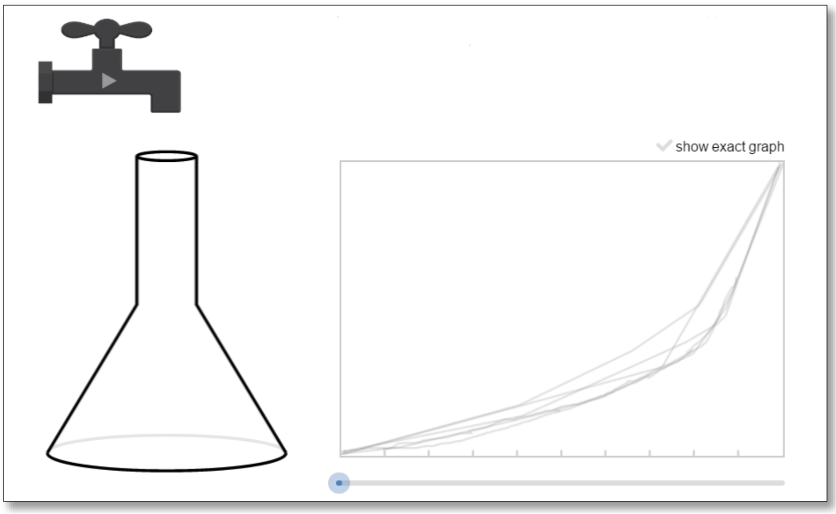

Figure 5.8 Water Line

5.4.6 The Bridge21 Model

Collaborating in teams is an element of the Bridge21 model. In the normal class students work in either pairs or groups of three and are encouraged to help one another. This depends on where they sit, as the tables are in twos and threes. Even though the students do not work alone in their normal class, working in groups emerged as strong theme in the group interviews. This may stem from the fact that there were set teams for the duration of the learning experience and how the activities were designed to challenge the students enough that no one individual would choose to do the activities on their own. Working in teams was seen as positive in an unspecified way (*f*=11) and beneficial to learning as it was easier to learn in a group or ideas could be shared to solve problems (*f*=27).

Other views emerged regarding working in teams. It was seen as negative in an unspecified way (*f*=1). There were times when it was felt that not enough of the work was done by all of the team (*f*=12). Some suggested it would have been better to make the groups smaller (*f*=4), the groups should be mixed up every so often (*f*=2) or the students should pick their own groups (*f*=1). From observation there were times when not every student was pulling his or her weight and there will always be some tension when working within a team. Some students learned more about working in teams and felt they improved in this area (*f*=9). This type of learning and the problem-solving nature of the learning experience would be in line with the Bridge21 model as it encourages the development of collaboration and problem-solving (Bridge21, 2017).

Each class began with a divergent thinking task. It took the participants a number of classes to get accustomed to these tasks. They helped with team formation in the first lesson and the students enjoyed them. No specific question was asked about this aspect of the learning experience but it was seen as positive (*f*=3).

The teams prepared a PowerPoint presentation on what they had learned during the learning experience and presented this during the final lesson. The students were not accustomed to doing this and while the presentations had some good elements to them they did not contain the depth of thinking about functions that was envisaged when this activity was planned. A specific question about this aspect of the experience was used in the group interviews. There were a wide range of thoughts expressed about the presentations. Maths was learned from either preparing or seeing the group presentations (f=11) with variety in the presentations being mentioned (f=4). The presentations were seen as positive (f=5) and some aspect of the group presentation was challenging e.g. putting it together (f=4).

Overall, the findings for the Bridge21 model were positive. This was to be expected given the positive findings in other applications of the model (Bray, 2015; Girvan et al., 2016).

5.4.7 Design of the Learning Experience

There were many positives to the learning experience outlined in the previous sections. The changes in the mean scores in the five subscales, while not all statistically significant, allied with the observational data of the researcher and the data from the group interviews shows that there was an increased level of engagement and confidence in those five areas. This change can be attributed to the learning experience which was designed in line with Bray's heuristics.

5.4.7.1 Variety and Depth

There were some expressions that the experience needed more variety towards the end. The researcher wanted depth within the topic of functions and chose to look at the topic of functions from many angles relating to the syllabus. From a mathematical viewpoint there was a lot of variety throughout the learning experience as students worked on moving between each of the multiple representations of functions from different starting points, used functions to model situations, learned about the effects of changing parameters of a function (Graham et al., 2010, p. 41) and learned about covariation. As well as this there were links to other area of the syllabus, such as algebra and statistics. An example of some of the depth within the content of the learning experience is shown in Figure 5.9 where linear functions and statistical data are linked.

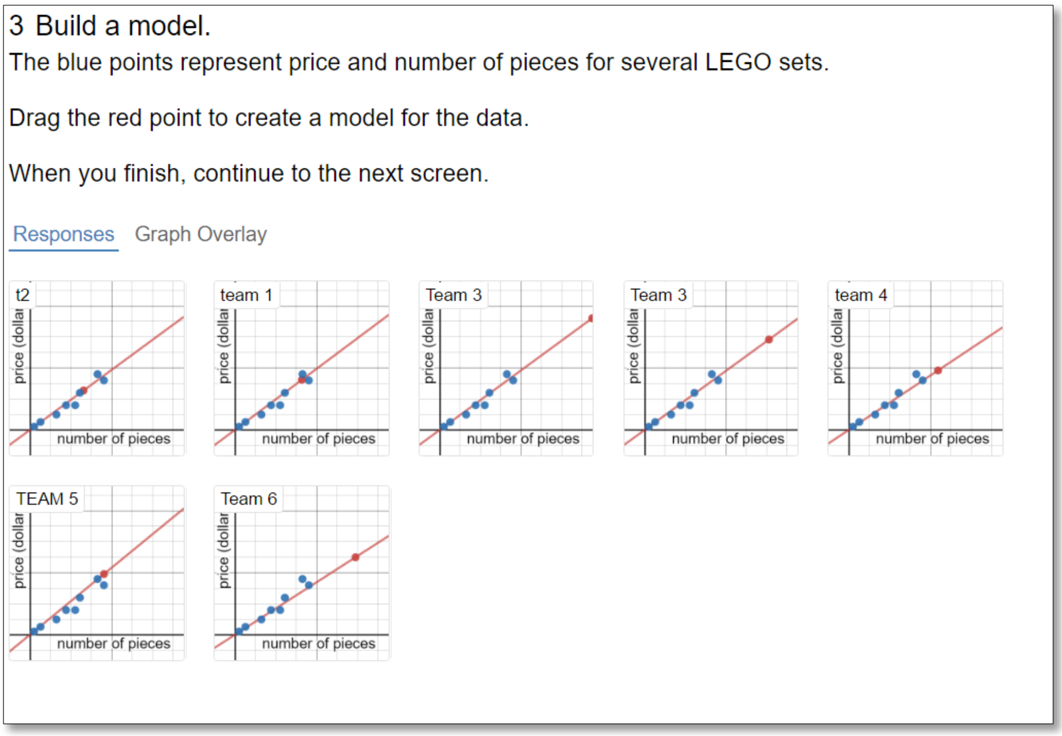

Figure 5.9 Fitting a Linear Function to Given Data

To encourage greater depth of understanding the teams were asked to explain some mathematical ideas. This would be a challenging type of question for this group. Figure 5.10 shows an example of what the teams wrote for a covariation task that asked them to describe what would happen the water's height vs. time graph for a cylinder that is narrower than the previous cylinder. There is a broad range in the answers and some are very expansive. This leads on to discussing the learning that occurred during the experience.

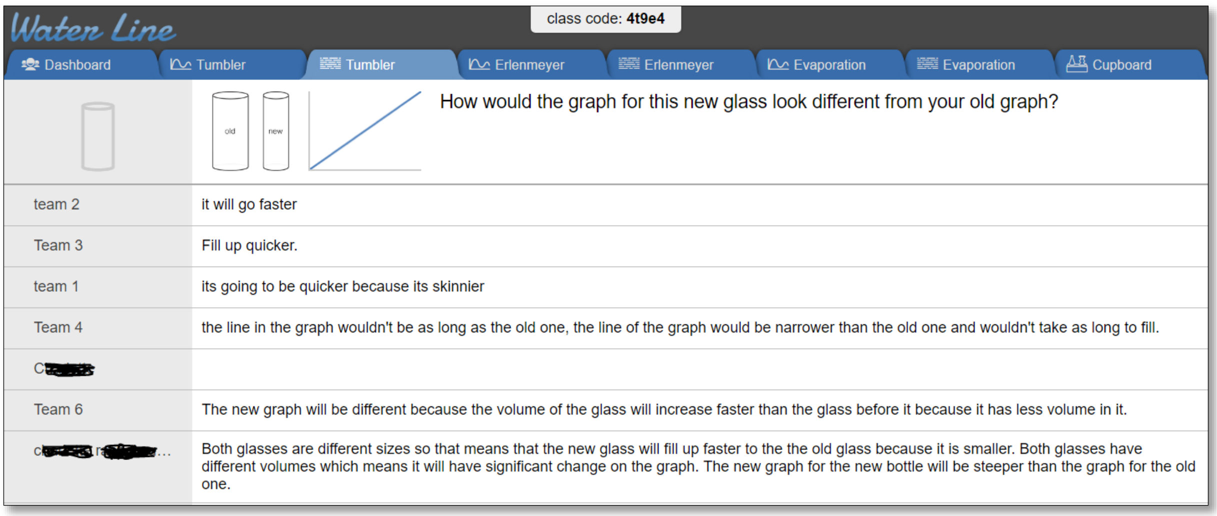

Figure 5.10 An Example of Written Answers

5.4.7.2 Learning

It was found that a lot of learning occurred during the learning experience. This finding is based on researcher observation, the data collected by the website and group interview data. The focus of the research is on attitudes and not improved conceptual understanding so no pre- and post- tests were conducted for student understanding in the area of functions. During the group interviews it was expressed that mathematics was learned (*f*=23), only a little maths was learned in the experience (*f*=2), no maths was learned (*f*=1) and learning (in an unspecified topic) occurred (*f*=2). Learning was also reported to have occurred through being a member of a team and the sharing ideas to solve problems (*f*=27). Aside from mathematics, learning about working in a group was reported to have occurred (*f*=9). Mathematics was also learned from the group presentations (*f*=11). It could also be argued that when students said they found some of the activities challenging (*f*=31), but could manage them (*f*=24) that mathematics learning occurred. The connection between learning, level of challenge and constructing learning together is illustrated in this extract from a group interview:

> *"TK: In what ways did you find working as part of a group helpful or unhelpful?*
>
> *2: It was helpful because you got to see everyone's kind of thinking like, you mightn't have known what to do but then they come up with a way to do it and you're like, oh yeah, that makes sense.*
>
> *TK: And here, what would you say?*
>
> *3: Yeah it was helpful, as I said before..*
>
> *4: You mightn't like know how a particular thing but someone else in the group probably would..*
>
> *1: It was helpful to get everyone's view on it so you could look at it.."*

5.4.7.3 Format

The participants of the group interviews were asked for suggestions of how to improve the learning experience. It was expressed that the format of the learning experience could be improved by having some classes in the normal classroom and some in the computer room (*f*=7). This suggestion came through strongly in the first two group interviews.

Group Interview 1:

> *"3: I think, like, kind of take a break from it, like, have like one class where you maybe explain what we had been doing or something like before we go and do it on the computers, like, to just explain it a bit more so it makes it kind of easier for us to do"*

Group Interview 2:

"4: I think it should have been every second like a normal class just to mix it up.

TK: There's a yeah coming from there. Would you agree with that?

2: Yes

For the final interview it was planned to ask specifically about this idea, if it didn't come up spontaneously in the suggestions from the participants.

Group Interview 3:

"TK: Do you want to add anything? Is there anything I haven't asked that you want to add about the experience?

2: I don't know, I'd love if we could like still go to the computer room

3: Yeah

2: and like stay here for one class and then move the next class go to the computer room

[general agreement]

and then come back

3: Yeah

1: Like everyone..

2: A change of scenery

TK: And do you think that set of lessons would have benefited from some classroom work?

1: Yeah definitely

3: Yeah if you merged them all, like, maybe just have one class a week in the computer room and then just the rest going over what you did and all that, but..."

5.5 Summary

The findings from the mixed methods approach used for this research provide evidence to answer the research question, which is can participation in a rich learning experience, designed in line with Bray's heuristics, improve student engagement and confidence when applied to the topic of functions within the confines of a normal school timetable? There were increases in the mean scores in all five subscales of the MTAS. Two of the subscales showed statistically significant improvements in attitudes. These were Affective Engagement (AE) and Attitude to Learning Mathematics with Technology (MT). The MTAS data on increases in attitude in these areas concurs with the observations of the researcher and the frequency of positive references made to these areas in the group interviews.

The other three subscales are Behavioural Engagement (BE), Confidence with Technology (TC) and Mathematical Confidence (MC). For BE the researcher observed a greater level of focus and perseverance during the learning experience from the majority of students the majority of the time in comparison to prior to the learning experience. There was also evidence from the group interviews that students felt they had to persevere during the learning experience. The positive behavioural engagement was displayed in the learning experience and commented on in the group interviews which points to the learning experience having some potential in this area.

TC was the subscale with the highest mean prior to the learning experience. The mean scores for TC increased from 14.3 to 15.8 which is indicative of some change in attitude. It was rare that help was required from the researcher during the experience for how to use the technology. The finding is that there were some marginal gains in TC as the participants using the relatively straight-forward technology.

MC was the subscale with, by far the lowest mean, 10.6, before the learning experience. The participants of the group interviews reported that mathematics was learned during the experience (f=23) and the researcher observed many moments where the students experienced success during the learning experience. These positive experiences with mathematical content had the potential to change the students' mathematical confidence. There was a mixture of responses when asked if the learning experience had changed their confidence in using and understanding maths, with no respondent saying it had decreased. The mean scores for MC increased from 10.6 to 11.5. Although the p-value of 0.270 shows that there was no significant improvement in this subscale there is a positive indication that the learning experience has potential in this area.

The findings for Bridge21, based only upon group interview data and researcher observation, were that the students benefited from collaborating in teams and the group presentations and found the divergent thinking tasks enjoyable and engaging.

A finding in relation to the design of the learning experience are that while the students found the experience engaging they felt it was more engaging at the start and needed more variation at the end. While there was a lot of diverse mathematical demands placed on students throughout the learning experience the students wanted greater diversity in other ways.

A second finding in relation to the design of the experience was that learning within the topic of functions and learning about working in groups took place through collaborating to solve multiple challenges.

Analysis of the group interview data revealed that the participants felt the learning experience could be improved by having some classes in the normal classroom and some in the computer room.

The purpose of this study was to build upon Bray's work to see if it was possible to use Bray's heuristics to design a technology-enhanced learning experience that can improve student engagement and confidence for content that is curriculum aligned within the confines of a normal school timetable. Overall, the main finding, from multiple sources, is that the learning experience can be attributed with improving student engagement and confidence, especially in two areas – affective engagement and attitude to learning mathematics with technology. There were also indications in the evidence that there were some small positive changes in the areas of behavioural engagement, confidence with technology and affective engagement.

6 Conclusions and Future Work

6.1 Introduction

This dissertation has successfully addressed the research question posed at the outset of this study. The research built on the work of Bray (Bray, 2015). A limitation of Bray's work was that it was conducted outside of the normal school timetable. This study has gone some way to addressing whether participation in a rich learning experience, designed in line with Bray's heuristics, can improve student engagement and confidence when applied to the topic of functions within the confines of a normal school timetable. The findings from many sources in this case study were used to answer the research question posed at the beginning of this investigation. There were statistically significant improvements in two areas – affective engagement and attitude to learning mathematics with technology. There were indications in the evidence that there were some small positive changes in the other areas of behavioural engagement, confidence with technology and affective engagement. These changes can be attributed to the learning experience. Bray's heuristics can be applied within the confines of normal school timetable and have a positive impact on student engagement and confidence.

The remainder of this chapter will outline the answer to the research question and present any interesting findings. The generalisability and limitations of the findings will be examined and possible areas for future research will be proposed.

6.2 Findings

The research question answered by this study is:

Can participation in a rich learning experience, designed in line with Bray's heuristics, improve student engagement and confidence when applied to the topic of functions within the confines of a normal school timetable?

The researcher observed clear positive differences in the behaviour relating to focus, enjoyment, and perseverance throughout the learning experience. An analysis of the group interview transcripts indicates that the students found the experience a positive, challenging and engaging one and there were also some views expressed relating to an increase in positive thoughts towards maths. The analysis of the pre- and post- learning experience data showed increases in the mean scores in all five subscales of the MTAS. Two of the subscales, Affective Engagement (AE) and Attitude to Learning Mathematics with Technology (MT) showed statistically significant improvements. These increases in positive attitudes can be attributed to the learning experience which was designed in line with Bray's heuristics and implemented within the confines of a normal school timetable. The answer to the research question is Bray's heuristics can be applied within the confines of normal school timetable and have a positive impact on student engagement and confidence.

Greater changes in the subscales were found in Bray's initial studies (Bray, 2015). There were statistically significant increases in all subscales for those interventions. For Bray's interventions that took place in school settings there were also increases in all subscales and significant increases in two subscales. It should be noted that these two subscales were the same as for this research i.e. AE and MT (Bray & Tangney, 2016). As described earlier, there were differences in the implementation of these studies to this one, including whether the group had experienced the Bridge21 model before and also that it was not conducted within normal school timetabled classes of 40 minutes or 80 minutes.

6.3 Generalisability

This study investigated one class group's experience for a short intervention of approximately 8 hours. Participating in the learning experience had a positive impact on this group's engagement and confidence. It is not possible to make any generalisations to the population as a whole from the findings from this one group. It may, however, be acceptable to make some tentative generalisations to groups that are similar to the one studied. While every class group and setting is different there are some characteristics of this group and this setting that are common elsewhere.

This group is one of two class groups in the school where the students will sit either the Leaving Certificate Ordinary Level or Foundation Level paper i.e. the lower 40% of students within the school. There are many other mixed-ability classes in Ireland with this characteristic, studying similar content within the confines of 40 and 80 minute classes. With one computer shared between a team of four and using freely available web based tools it means similar technology-enhanced learning experiences can be replicated across Ireland. Given that functions are included in many syllabi in other countries similarly designed experiences could be used in other countries too.

6.4 Limitations

This study used one group for a short intervention to investigate the research question. There are many differences between any two groups that could potentially influence a set of findings. Functions is a large area in mathematics. An investigation of a greater number of class groups, studying more areas within functions would have benefitted the reliability of the findings.

The group interviews were conducted by the researcher, which may have led to bias in the answers given by the participants. To mitigate against any bias caused by this the researcher asked the interviewees to be honest and give their own opinion. This was not ideal, but was adequate for the purposes of acquiring rich data pertinent to answering the research question. The alternative of having someone unfamiliar with the learning experience to conduct the interview was considered, but this would have meant this individual would have been less likely to probe further into anything relevant that turned up in the course of an interview.

Another limitation regarding the group interviews was the researcher is not an experienced interviewer. A balance has to be struck between making the participants comfortable to answer questions and the skill of probing deeper into what they are saying. From listening to the recordings and analysing the transcript data several avenues of further exploration of points raised were missed due to the researcher being an inexperienced interviewer.

The qualitative data collected throughout the learning experience was the field notes made by the researcher. The research would have benefitted from having an additional observer collecting data about the learning experience in a more systematic way.

Given the frequency of comments about how different the learning experience was from the norm for this group a limitation of this study is that novelty could have been a confounding factor in the results. This point will be elaborated upon in the next section, which discusses recommendations for future work in this research area.

6.5 Future Directions

- Bray's participants had experienced the Bridge21 model before her interventions and she felt they may have been positively disposed to the methodology and this may have impacted the results (Bray, 2015). Novelty emerged as a theme from the group interviews in this study. The group for this study had neither used computers for learning mathematics in the past nor had experienced the Bridge21 model. It would be interesting to find a group that has used computers in a non-transformative manner for mathematics in the past and then have them experience an intervention based on Bray's heuristics. Using such a group would mitigate against any novelty attached to using computers and mitigate against having students that have been positively disposed to the Bridge21 model.

- The cohort of students investigated had low mathematical confidence. Differing groups could be investigated to see if similar or better results can be found. These groups could include, or exclusively, consist of students that are planning on sitting the Leaving Certificate Higher Level paper.

- Bray recommended more in-school interventions to see whether her findings could be "replicated - both for repeated use with similar students, and for greater numbers of classes following syllabi leading to state examinations" (2015, p. 172). This study used content that can be examined in state examinations for a learning experience with a duration of 13 class periods. It would be worth exploring the influence of longer learning experiences that are implemented within the confines of a normal school timetable on student engagement and confidence.

- The participants suggested that to improve upon this positive learning experience there could be some classes that do not involve using computers after some classes using computers. For example, the students could spend four or five classes exploring and problem-solving using a computer and then there could be one class in a normal classroom used to consolidate and apply what was learned using pen and paper.

6.6 Summary

This dissertation applied Bray and Tangney's work of classifying recent studies of technology interventions in mathematics education to three recent papers on using technology to teach functions.

This case study has demonstrated that it is possible to design and implement a rich learning experience, designed in line with Bray's heuristics, in order to improve student engagement and confidence in the topic of functions within the confines of a normal school timetable. Possible areas for future research were also identified.

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Appendix 1: The Desmos Guide to Building Great (Digital) Math Activities

We wrote an activity building code for two reasons:

People have asked us what Desmos pedagogy looks like. They've asked about our values.

We spend a lot of our work time debating the merits and demerits of different activities and we needed some kind of guide for those conversations beyond our individual intuitions and prejudices.

So the Desmos Faculty – Shelley Carranza, Christopher Danielson, Michael Fenton, Dan Meyer – wrote this guide. It has already improved our conversations internally. We hope it will improve our conversations *externally* as well, with the broader community of math educators we're proud to serve.

NB. We work with digital media but we think these recommendations apply pretty well to print media also.

Incorporate a variety of verbs and nouns. An activity becomes tedious if students do the same kind of verb over and over again (calculating, let's say) and that verb results in the same kind of noun over and over again (a multiple choice response, let's say). So attend to the verbs you're assigning to students. Is there a variety? Are they calculating, but also arguing, predicting, validating, comparing, etc? And attend to the kinds of nouns those verbs produce. Are students producing numbers, but also representing those numbers on a number line and writing sentences about those numbers?

Ask for informal analysis before formal analysis. Computer math tends to emphasize the most formal, abstract, and precise mathematics possible. We know that kind of math is powerful, accurate, and efficient. It's also the kind of math that computers are well-equipped to assess. But we need to access and promote a student's informal understanding of mathematics also, both as a means to interest the student in more formal mathematics *and* to prepare her to *learn* that formal mathematics. So ask for estimations before calculations. Conjectures before proofs. Sketches before graphs. Verbal rules before algebraic rules. Home language before school language.

Create an intellectual need for new mathematical skills. Ask yourself, "Why did a mathematician invent the skill I'm trying to help students learn? What problem were they trying to solve? How did this skill make their intellectual life easier?" Then ask yourself, "How can I help students experience that need?" We calculate because calculations offer more certainty than estimations. We use variables so we don't have to run the same calculation over and over again. We prove because we want to settle some doubt. Before we offer the aspirin, we need to make sure students are experiencing a headache.

Create problematic activities. A problematic activity feels *focused* while a problem-free activity *meanders*. A problem-free activity picks at a piece of mathematics and asks lots of small questions about it, but the larger frame for those smaller questions isn't apparent. A problem-free task gives students a parabola and then asks questions about its vertex, about its line of symmetry, about its intercepts, simply because it *can* ask those questions, not because it *must*. Don't create an activity with lots of small pieces of analysis at the start that are only clarified by some larger problem later. Help us understand why we're here. Give us the larger problem now.

Give students opportunities to be right and wrong in different, interesting ways. Ask students to sketch the graph of a linear equation, but also ask them to sketch *any* linear equation that has a positive slope and a negative y-intercept. Thirty correct answers to that second question will illuminate mathematical ideas that thirty correct answers to the first question will not. Likewise, the number of interesting ways a student can answer a question incorrectly signals the value of the question as formative assessment.

Delay feedback for reflection, especially during concept development activities. A student manipulates one part of the graph and another part changes. If we ask students to change the first part of the graph so the second reaches a particular target value or coordinate, it's possible – even likely – the student will complete the task through guess-and-check, without thinking mathematically at all. Instead, delay that feedback briefly. Ask the student to reflect on where the first part of the graph should be so the second will hit the target. Then ask the student to check her prediction on a *subsequent* screen. That interference in the feedback loop may restore reflection and meta-cognition to the task.

Connect representations. Understanding the connections between representations of a situation – tables, equations, graphs, and contexts – helps students understand the representations themselves. In a typical word problem, the student converts the context into a table, equation, or graph, and then translates between those three formats, leaving the context behind. (Thanks, context! Bye!) The digital medium allows us to re-connect the math to the context. You can see how changing your equation changes the parking lines. You can see how changing your graph changes the path of the Cannon Man. "And in any case joy in being a cause is well-nigh universal."

Create objects that promote mathematical conversations between teachers and students. Create perplexing situations that put teachers in a position to ask students questions like, "What if we changed this? What would happen?" Ask questions that will generate arguments and conversations that the teacher can help students settle. Maximize the ratio of conversation time per screen, particularly in concept development activities. All other things being equal, fewer screens and inputs are better than more. If one screen is extensible and interesting enough to support ten minutes of conversation, ring the gong.

Create cognitive conflict. Ask students for a prediction – perhaps about the trajectory of a data set. If they feel confident about that prediction and it turns out to be wrong, that alerts their brain that it's time to shrink the gap between their prediction and reality, which is "learning," by another name. Likewise, aggregate student thinking on a graph. If students were convinced the answer is obvious and shared by all, the fact that there is widespread disagreement may provoke the same readiness.

Keep expository screens short, focused, and connected to existing student thinking. Students tend to ignore screens with paragraphs and paragraphs of expository text. Those screens may connect poorly, also, to what a student already knows, making them ineffective even if students pay attention. Instead, add that exposition to a teacher note. A good teacher has the skill a computer lacks to determine what subtle connections she can make between a student's existing conceptions to the formal mathematics. Or, try to use computation layer to refer back to what students already think, incorporating and responding to those thoughts in the exposition. (eg. "On screen 6, you thought the blue line would have the greater slope. Actually, it's the red line. Here's how you can know for sure next time.")

Integrate strategy and practice. Rather than just asking students to solve a practice set, also ask those students to decide in advance which problem in the set will be hardest and why. Ask them to decide before solving the set which problem will produce the largest answer and how they know. Ask them to create a problem that will have a larger answer than any of the problems given. This technique raises the ceiling on our definition of "mastery" and it adds more dimensions to a task – practice – that often feels unidimensional.

Create activities that are easy to start and difficult to finish. Bad activities are too difficult to start and too easy to finish. They ask students to operate at a level that's too formal too soon and then they grant "mastery" status after the student has operated at that level after some small amount of repetition. Instead, start the activity by inviting students' informal ideas and then make mastery hard to achieve. Give advanced students challenging tasks so teachers can help students who are struggling.

Ask proxy questions. Would I use this with my own students? Would I recommend this if someone asked if we had an activity for that mathematical concept? Would I check out the laptop cart and drag it across campus for this activity? Would I want to put my work from this activity on a refrigerator? Does this activity generate delight? How much better is this activity than the same activity on paper?

Appendix 2: Mapping the Activities to Bray's Heuristics

Name of activity: Water Line https://teacher.desmos.com/waterline

Description: Teams draw the height of the water in various glasses against time. They can also see how accurate their graph is by seeing the actual water level being compared to the water level of their graph. To finish teams create their own glass and all teams must draw the height of the water against time.

Type(s) of thinking: Covariation

Curriculum content: Relations without formulae.

− using graphs to represent phenomena quantitatively.

− make sense of quantitative graphs and draw conclusions from them

− make connections between the shape of a graph and the story of a phenomenon

− describe both quantity and change of quantity on a graph

Related Student Difficulty:

Bray's Heuristics used:

1.	(a) Team-based	✓
	(b) encourages collaboration,	✓
	(c) socially constructivist approach to learning	✓
2.	(a) Exploit the transformative capabilities of technology	✓
	(b) Exploit computational capabilities of the technology.	
3.	Makes use of a variety of technologies (digital and traditional)	✓
4.	involve problem-solving, investigation and sense-making,	✓
	involve guided discovery,	✓
	be situated in a meaningful/real context,	✓
	move from concrete to abstract concepts,	✓
	be open-ended but with constraints,	✓
	be cross-curricular/cross-strand,	
	be focused on skill development as well as on content,	
	have a 'low-floor' and a 'high-ceiling'.	✓
5.	21st Century Learning and activity design	✓

Name of activity: Function Carnival https://teacher.desmos.com/carnival

Description: Teams sketch graphs to show how a variable changes against time.

Type(s) of thinking: Covariation

Curriculum content: Relations without formulae

– using graphs to represent phenomena quantitatively

– explore graphs of motion

– make sense of quantitative graphs and draw conclusions from them

– make connections between the shape of a graph and the story of a phenomenon

– describe both quantity and change of quantity on a graph

Related Student Difficulty: Distinguishing between visual characteristics of a situation and similar characteristics of the graph of a function that models the situation (Carlson et al., 2005).

Additional info: Desmos describe why they made this activity for reasons similar to the misconception above (Desmos, 2014).

Bray's Heuristics used:

1.	(a) Team-based	✓
	(b) encourages collaboration,	✓
	(c) socially constructivist approach to learning	✓
2.	(a) Exploit the transformative capabilities of technology	✓
	(b) Exploit computational capabilities of the technology.	✓
3.	Makes use of a variety of technologies (digital and traditional)	✓
4.	involve problem-solving, investigation and sense-making,	✓
	involve guided discovery,	✓
	be situated in a meaningful/real context,	✓
	move from concrete to abstract concepts,	
	be open-ended but with constraints,	
	be cross-curricular/cross-strand,	✓
	be focused on skill development as well as on content,	
	have a 'low-floor' and a 'high-ceiling'.	✓
5.	21st Century Learning and activity design	✓

Name of activity: Function Carnival, Part 2

https://teacher.desmos.com/activitybuilder/custom/56d7528a9e6961430932681e

Description: Teams analyse some graphs from Function Carnival in more detail. They relate points on the graph describe in words with function notation for the points and identify the advantages of each representation. They are also challenged by questions that show how many differing inputs could have the one output.

Type(s) of thinking: Multiple representations of functions, solving and substituting.

Curriculum content:

− make sense of quantitative graphs and draw conclusions from them

− make connections between the shape of a graph and the story of a phenomenon

– use graphical methods to find approximate solutions to $f(x) = k$

Related Student Difficulty: Understanding functions in terms of input and output can be a major challenge for most students (Carlson et al., 2005).

Bray's Heuristics used:

1.	(a) Team-based	✓
	(b) encourages collaboration,	✓
	(c) socially constructivist approach to learning	✓
2.	(a) Exploit the transformative capabilities of technology	✓
	(b) Exploit computational capabilities of the technology.	✓
3.	Makes use of a variety of technologies (digital and traditional)	✓
4.	involve problem-solving, investigation and sense-making,	✓
	involve guided discovery,	✓
	be situated in a meaningful/real context,	✓
	move from concrete to abstract concepts,	
	be open-ended but with constraints,	
	be cross-curricular/cross-strand,	✓
	be focused on skill development as well as on content,	
	have a 'low-floor' and a 'high-ceiling'.	✓
5.	21st Century Learning and activity design	✓

Name of activity: Lego Prices https://teacher.desmos.com/activitybuilder/custom/57e563aa072703f509160cc2

Description: Teams make a prediction for the price of a large box of Lego with very limited information. They then sketch the relationship between price and number of Lego pieces. They build a linear model using a line of best fit and use their model to make a more precise prediction. They explain the parameters of their model in context and make a prediction about the number of pieces in an expensive box of Lego.

Type(s) of thinking: Model situations with functions (Graham et al., 2010, p. 41).

Curriculum content:

− determine the relationship between variables using scatterplots

− generalise and explain patterns and relationships in words and numbers

− find the underlying formula algebraically from which the data are derived (linear, quadratic relations)

− discuss rate of change and the *y*-intercept; consider how these relate to the context from which the relationship is derived, and identify how they can appear in a table, in a graph and in a formula

− devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

– use graphical methods to find approximate solutions to $f(x) = k$

Related Student Difficulties: Viewing functions as two expressions separated by an equals sign (Thompson, 1994).

Difficulties in forming a function when given a function situation described in words.

Bray's Heuristics used:

1.	(a) Team-based	✓
	(b) encourages collaboration,	✓
	(c) socially constructivist approach to learning	✓
2.	(a) Exploit the transformative capabilities of technology	✓
	(b) Exploit computational capabilities of the technology.	✓
3.	Makes use of a variety of technologies (digital and traditional)	✓
4.	involve problem-solving, investigation and sense-making,	✓
	involve guided discovery,	✓
	be situated in a meaningful/real context,	✓
	move from concrete to abstract concepts,	
	be open-ended but with constraints,	
	be cross-curricular/cross-strand,	✓
	be focused on skill development as well as on content,	✓
	have a 'low-floor' and a 'high-ceiling'.	✓
5.	21st Century Learning and activity design	✓

Name of activity: Polygraph Linear Functions 1

Description: Teams identify a graph chosen by another team by asking questions that can only be answered Yes or No. The questions should relate to the properties of linear functions i.e. slope, y-intercept and domain. The purpose is to develop greater precision in language.

Type(s) of thinking: Model situations with functions (Graham et al., 2010, p. 41).

Curriculum content:

− engage with the concept of a function, domain, co-domain and range

− investigate relations of the form $y = mx$ and $y = mx + c$

Related Student Difficulties: Thinking that constant functions (e.g., y = 5) are not functions because they do not vary.

Bray's Heuristics used:

1.	(a) Team-based	✓
	(b) encourages collaboration,	✓
	(c) socially constructivist approach to learning	✓
2.	(a) Exploit the transformative capabilities of technology	
	(b) Exploit computational capabilities of the technology.	
3.	Makes use of a variety of technologies (digital and traditional)	✓
4.	involve problem-solving, investigation and sense-making,	✓
	involve guided discovery,	✓
	be situated in a meaningful/real context,	
	move from concrete to abstract concepts,	✓
	be open-ended but with constraints,	
	be cross-curricular/cross-strand,	✓
	be focused on skill development as well as on content,	✓
	have a 'low-floor' and a 'high-ceiling'.	✓
5.	21st Century Learning and activity design	✓

Name of activity: Marbleslides 1

Description: Marbles slide along linear functions to hit stars. Teams change the properties of the linear functions (slope, y-intercept and domain) and create new linear functions so the marbles hit all the stars.

Type(s) of thinking: Multiple representations of functions and analysing the effects of parameters (Graham et al., 2010, p. 41).

Curriculum content:

− engage with the concept of a function, domain, co-domain and range

− investigate relations of the form $y = mx$ and $y = mx + c$

− find the underlying formula algebraically from which the data are derived (linear, quadratic relations)

Related Student Difficulties:

Bray's Heuristics used:

1.	(a) Team-based	✓
	(b) encourages collaboration,	✓
	(c) socially constructivist approach to learning	✓
2.	(a) Exploit the transformative capabilities of technology	✓
	(b) Exploit computational capabilities of the technology.	
3.	Makes use of a variety of technologies (digital and traditional)	✓
4.	involve problem-solving, investigation and sense-making,	✓
	involve guided discovery,	✓
	be situated in a meaningful/real context,	✓
	move from concrete to abstract concepts,	✓
	be open-ended but with constraints,	✓
	be cross-curricular/cross-strand,	
	be focused on skill development as well as on content,	✓
	have a 'low-floor' and a 'high-ceiling'.	✓
5.	21st Century Learning and activity design	✓

Name of activity: Distance Time Graphs

Description: Teams draw distance time graphs to represent some stories and explain the misconceptions in an incorrect graph.

Type(s) of thinking: Model situations with functions (Graham et al., 2010) and covariation (Carlson et al., 2002)

Curriculum content:

− relations without formulae

− using graphs to represent phenomena quantitatively

− explore graphs of motion

− make sense of quantitative graphs and draw conclusions from them

− make connections between the shape of a graph and the story of a phenomenon

− describe both quantity and change of quantity on a graph

Related Student Difficulties:

Bray's Heuristics used:

1.	(a) Team-based	✓
	(b) encourages collaboration,	✓
	(c) socially constructivist approach to learning	✓
2.	(a) Exploit the transformative capabilities of technology	
	(b) Exploit computational capabilities of the technology.	
3.	Makes use of a variety of technologies (digital and traditional)	✓
4.	involve problem-solving, investigation and sense-making,	✓
	involve guided discovery,	✓
	be situated in a meaningful/real context,	✓
	move from concrete to abstract concepts,	
	be open-ended but with constraints,	✓
	be cross-curricular/cross-strand,	
	be focused on skill development as well as on content,	
	have a 'low-floor' and a 'high-ceiling'.	✓
5.	21st Century Learning and activity design	✓

Name of activity: Playing Catch-Up https://teacher.desmos.com/activitybuilder/custom/5818fb314e762b653c3bf0f3

Description: Information about a 40 yard race between two people is given through video. One runner is slow and the other is very fast. Three types of races are shown (with factors being changed, such as one runner getting a head start and the speed of one runner being halved). From this students must make an initial prediction as to when both runners will be at the same point and draw their own distance time graph for both runners for a race where one runner gets a 1 second head start and the other runs at half speed. Then make more accurate predictions are made as more information is given through a partially filled in distance time graph, a partially filled in table of times and distances and with algebraic functions for the distance travelled of both runners. At the end of the activity students were asked create their own story for a race, which they had to graph, create a table for and also create an algebraic function for.

Type(s) of thinking: Multiple representations of functions, modelling situations with functions (Graham et al., 2010) and covariation (Carlson et al., 2002)

Curriculum content:

− use the representations to reason about the situation from which the relationship is derived and communicate their thinking to others

− using graphs to represent phenomena quantitatively

− use graphical methods to find approximate solutions to $f(x) = g(x)$

− select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: $f(x) = g(x)$, with $f(x) = ax + b$, $g(x) = cx + d$ where $a, b, c, d \in \mathbb{Q}$

Related Student Difficulties:

Bray's Heuristics used:

1.	(a) Team-based	✓
	(b) encourages collaboration,	✓
	(c) socially constructivist approach to learning	✓
2.	(a) Exploit the transformative capabilities of technology	✓
	(b) Exploit computational capabilities of the technology.	
3.	Makes use of a variety of technologies (digital and traditional)	✓
4.	involve problem-solving, investigation and sense-making,	✓
	involve guided discovery,	
	be situated in a meaningful/real context,	✓
	move from concrete to abstract concepts,	✓
	be open-ended but with constraints,	✓
	be cross-curricular/cross-strand,	✓
	be focused on skill development as well as on content,	✓
	have a 'low-floor' and a 'high-ceiling'.	✓
5.	21st Century Learning and activity design	✓

Name of activity: Match My Linear Function

Description: Teams are faced with a series of graphing challenges designed to build student understanding of linear functions. Functions notation for functions and points on functions is used throughout. The teams are also asked to explain their thinking on a number of occasions.

Type(s) of thinking: Multiple representations of functions and analysing the effects of parameters (Graham et al., 2010).

Curriculum content:

− engage with the concept of a function, domain, co-domain and range

− investigate relations of the form $y = mx$ and $y = mx + c$

− find the underlying formula algebraically from which the data are derived (linear, quadratic relations)

Related Student Difficulties: Viewing functions as two expressions separated by an equal sign (Thompson, 1994).

Bray's Heuristics used:

1.	(a) Team-based	✓
	(b) encourages collaboration,	✓
	(c) socially constructivist approach to learning	✓
2.	(a) Exploit the transformative capabilities of technology	✓
	(b) Exploit computational capabilities of the technology.	
3.	Makes use of a variety of technologies (digital and traditional)	✓
4.	involve problem-solving, investigation and sense-making,	✓
	involve guided discovery,	
	be situated in a meaningful/real context,	✓
	move from concrete to abstract concepts,	✓
	be open-ended but with constraints,	
	be cross-curricular/cross-strand,	✓
	be focused on skill development as well as on content,	✓
	have a 'low-floor' and a 'high-ceiling'.	✓
5.	21st Century Learning and activity design	✓

Name of activity: Charge! https://teacher.desmos.com/activitybuilder/custom/563a59893f80f2fd0b7c77f0

Description: Teams use limited information to create a linear model to predict how long it will take for a phone to fully charge. The teams also have to interpret the parameters of their model in context.

Type(s) of thinking: Modelling and covariation

Curriculum content:

− find the underlying formula algebraically from which the data are derived (linear, quadratic relations)

− investigate relations of the form $y = mx$ and $y = mx + c$

– use graphical methods to find approximate solutions to $f(x) = k$

− select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: $f(x) = g(x)$, with $f(x) = ax + b$, $g(x) = cx + d$ where $a, b, c, d \in \mathbb{Q}$

Related Student Difficulties: Understanding functions in terms of input and output can be a major challenge for most students (Carlson et al., 2005). Viewing functions as two expressions separated by an equals sign (Thompson, 1994).

Bray's Heuristics used:

1.	(a) Team-based	✓
	(b) encourages collaboration,	✓
	(c) socially constructivist approach to learning	✓
2.	(a) Exploit the transformative capabilities of technology	✓
	(b) Exploit computational capabilities of the technology.	✓
3.	Makes use of a variety of technologies (digital and traditional)	✓
4.	involve problem-solving, investigation and sense-making,	✓
	involve guided discovery,	
	be situated in a meaningful/real context,	✓
	move from concrete to abstract concepts,	
	be open-ended but with constraints,	
	be cross-curricular/cross-strand,	✓
	be focused on skill development as well as on content,	
	have a 'low-floor' and a 'high-ceiling'.	✓
5.	21^{st} Century Learning and activity design	✓

Name of activity: T-Shirt Offers

Description: Teams are given information from two companies with different fixed and variable costs for T-Shirts. They must work out which offer is best in given situations, read and interpret common values from graphs and solve the problem algebraically. Following this the underlying concepts of intersecting functions are explored further and students are asked to generalise how two functions $f(x) = 3x + 10$ and $g(x) = ax + b$ will have a point of intersection.

Type(s) of thinking: Multiple representations of functions, modelling situations with functions and analysing the effects of parameters.

Curriculum content:

− find the underlying formula algebraically from which the data are derived (linear, quadratic relations)

− use the representations to reason about the situation from which the relationship is derived and communicate their thinking to others

− using graphs to represent phenomena quantitatively

− select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: $f(x) = g(x)$, with $f(x) = ax + b$, $g(x) = cx + d$ where $a, b, c, d \in \mathbb{Q}$

− decide if two linear relations have a common value

Related Student Difficulties: Distinguishing between an algebraically defined function and an equation. Forming a function when given a function situation described in words

Bray's Heuristics used:

1.	(a) Team-based	✓
	(b) encourages collaboration,	✓
	(c) socially constructivist approach to learning	✓
2.	(a) Exploit the transformative capabilities of technology	✓
	(b) Exploit computational capabilities of the technology.	
3.	Makes use of a variety of technologies (digital and traditional)	✓
4.	involve problem-solving, investigation and sense-making,	✓
	involve guided discovery,	
	be situated in a meaningful/real context,	✓
	move from concrete to abstract concepts,	✓
	be open-ended but with constraints,	
	be cross-curricular/cross-strand,	✓
	be focused on skill development as well as on content,	✓
	have a 'low-floor' and a 'high-ceiling'.	✓
5.	21^{st} Century Learning and activity design	✓

Appendix 3: Research Ethics Committee Approval

TCD REC WebApp: The status of 'An Investigation into Bray's Heuristics for Mathematics Learning Activities as Applied to Functions' (180) has been updated by the Committee Inbox x

rec-app-help@tchpc.tcd.ie

to me

3 Feb

The status of 'An Investigation into Bray's Heuristics for Mathematics Learning Activities as Applied to Functions' has been updated by the Committee.

Title: 'An Investigation into Bray's Heuristics for Mathematics Learning Activities as Applied to Functions'
Applicant Name: Tony Knox
Submitted by: Tony Knox
Academic Supervisor: Brendan Tangney
Application Number: 20161202

Result of the REC Meeting: Approved

The Feedback from the Committee is as follows:
The revised application appears to adequately address the comments from earlier review.

The application can be viewed here:

https://webhost.tchpc.tcd.ie/research_ethics/?q=node/180

Appendix 4: Information Sheets, Consent and Assent Forms

Parent(s)/Guardian(s)
Information Sheet

TITLE OF PROJECT:
An Investigation into Bray's Heuristics for Mathematics Learning Activities as Applied to Functions

LEAD RESEARCHER: Tony Knox

BACKGROUND TO RESEARCH:
The research is part of the project work for a postgraduate degree in Technology and Learning in Trinity College Dublin, the University of Dublin, and may be included in related research publications.

The research examines the usefulness of providing students with mathematics problems that will be investigated using a website called Desmos. The research seeks to find out if students engaging with these problems with the software will lead the students to learn many properties of functions and also if their attitude to mathematics will change.

PROCEDURES OF THIS STUDY:
As part of this study participants will be asked to:
1. Complete a fully anonymised questionnaire before the study.
2. Participate in a series of twelve, 40-minute, classes during their timetabled mathematics classes, engaging with some mathematics problems using the Desmos website.
3. Complete a fully anonymised questionnaire after the study.
4. Participate in a 20-minute group interview to assess their experience of the study. I understand this interview will be recorded (audio only) and once transcribed, the audio recording will be deleted. Only the researcher will have access to this recording until the recording is destroyed.

PARTICIPATION:
The learning activities are directly relevant to the class and cover learning outcomes from the students' mathematics syllabus. All students in the class will participate in the learning activities (number 2. above), as normal. While students cannot opt out of the learning activity they can at any stage exclude themselves from the research aspects of the activity (numbers 1., 3. and 4. above).

ILLICIT ACTIVITY:
In the unlikely event that illicit activities become known over the course of this research, these will be reported to appropriate authorities.

DATA RECORDING:
Data collected will be separated from personal identity information as soon as possible after collection. Codes will be used to identify individual cases. The key linking such codes to personal information - names will be kept secure and separate from the dataset.

CONFLICT OF INTEREST:
Participants include students in the school where I am employed.

PUBLICATION:
The results from this study will be presented as part of the project work for a postgraduate degree in Technology and Learning in Trinity College Dublin, the University of Dublin, and may be included in related research publications.

DECLARATION:

- I am 18 years or older and am competent to provide consent.
- I have read, or had read to me, a document providing information about this research and this consent form. I have had the opportunity to ask questions and all my questions have been answered to my satisfaction and understand the description of the research that is being provided to me.
- I agree that my data is used for scientific purposes and I have no objection that my data is published in scientific publications in a way that does not reveal my identity.
- I understand that if I make illicit activities known, these will be reported to appropriate authorities.
- I understand that I may stop electronic recordings at any time, and that I may at any time, even subsequent to my participation have such recordings destroyed (except in situations such as above).
- I understand that, subject to the constraints above, no recordings will be replayed in any public forum or made available to any audience other than the current researchers/research team.
- I freely and voluntarily agree to be part of this research study, though without prejudice to my legal and ethical rights.
- I understand that I may refuse to answer any question and that I may withdraw at any time without penalty.
- I understand that my participation is fully anonymous and that no personal details about me will be recorded.
- I understand that if I or anyone in my family has a history of epilepsy then I am proceeding at my own risk.
- I have received a copy of this agreement.

Parent(s)/Guardian(s) Consent Form

DECLARATION:

- I am 18 year or over and competent to provide consent.
- I have read, or had read to me, an information form providing information about this research (as detailed in the information sheet) and this consent form.
- I understand that my son's/daughter's participation is fully anonymous and that no personal details about him/her will be recorded.
- I understand that it is a staff member of [redacted] College running this study but that no information in this study will be used to identify my son/daughter.
- I have had the opportunity to ask questions and all my questions have been answered to my satisfaction. I understand the description of the research that is being provided to me.
- I agree to my son's/daughter's data being presented as part of the project work for the MSc in Technology and Learning (TCD) in a way that does not reveal his/her identity.
- I freely and voluntarily agree to my son/daughter being part of this research study, though without prejudice to her legal and ethical rights.
- I understand that he/she may refuse to answer any question and that he/she may withdraw at any time without penalty.
- I consent to him/her being observed, by the researcher through note-taking, while completing the tasks associated with this project.
- I understand that in the unlikely event that illicit activities become known over the course of this research, these will be reported to appropriate authorities.
- I understand that his/her data will be stored securely and deleted on completion of the study.
- I understand that the study involves viewing a computer screen and that if my son/daughter or anyone in his/her family has a history of epilepsy then he/she is proceeding at his/her own risk.
- I have received a copy of this agreement.

I ________________________ consent to my son/daughter ____________________ taking part in this research project.

Signature of Parent/Guardian: ___________________________ Date: ___________

Signature of project leader (TCD): __ Brendan Tangney _____ Date: __04/Dec____

Statement of investigator's responsibility:
I have explained the nature and purpose of this research study, the procedures to be undertaken and any risks that may be involved. I have offered to answer any questions and fully answered such questions. I believe that the participant understands my explanation and has freely given informed consent. I undertake to act in accordance with the information supplied.

RESEARCHER: Tony Knox tonyknox@[redacted]
SUPERVISOR: Brendan Tangney tangey@tcd.ie

Student Participant
Information Sheet

TITLE OF PROJECT:
An Investigation into Bray's Heuristics for Mathematics Learning Activities as Applied to Functions

LEAD RESEARCHER: Tony Knox

BACKGROUND TO RESEARCH:
The research is part of the project work for a postgraduate degree in Technology and Learning in Trinity College Dublin, the University of Dublin, and may be included in related research publications.

The research examines the usefulness of providing students with mathematics problems that will be investigated using a website called Desmos. The research seeks to find out if students engaging with these problems with the software will lead the students to learn many properties of functions and also if their attitude to mathematics will change.

PROCEDURES OF THIS STUDY:
As part of this study participants will be asked to:

1. Complete a fully anonymised questionnaire before the study.
2. Participate in a series of twelve, 40-minute, classes during their timetabled mathematics classes, engaging with some mathematics problems using the Desmos website.
3. Complete a fully anonymised questionnaire after the study.
4. Participate in a 20-minute group interview to assess their experience of the study. I understand this interview will be recorded (audio only) and once transcribed, the audio recording will be deleted. Only the researcher will have access to this recording until the recording is destroyed.

PARTICIPATION:
The learning activities are directly relevant to the class and cover learning outcomes from the students' mathematics syllabus. All students in the class will participate in the learning activities (number 2. above), as normal. While students cannot opt out of the learning activity they can at any stage exclude themselves from the research aspects of the activity (numbers 1., 3. and 4. above).

ILLICIT ACTIVITY:
In the unlikely event that illicit activities become known over the course of this research, these will be reported to appropriate authorities.

DATA RECORDING:
Data collected will be separated from personal identity information as soon as possible after collection. Codes will be used to identify individual cases. The key linking such codes to personal information - names will be kept secure and separate from the dataset.

CONFLICT OF INTEREST:
Participants include students in the school where I am employed.

PUBLICATION:
The results from this study will be presented as part of the project work for a postgraduate degree in Technology and Learning in Trinity College Dublin, the University of Dublin, and may be included in related research publications.

DECLARATION:

- I assent to participate in this research. As I am under 18, I understand that permission must also be provided by my parent/guardian to participate in this study.
- I have read, or had read to me, a document providing information about this research and this consent form. I have had the opportunity to ask questions and all my questions have been answered to my satisfaction and understand the description of the research that is being provided to me.
- I agree that my data is used for scientific purposes and I have no objection that my data is published in scientific publications in a way that does not reveal my identity.
- I understand that if I make illicit activities known, these will be reported to appropriate authorities.
- I understand that I may stop electronic recordings at any time, and that I may at any time, even subsequent to my participation have such recordings destroyed (except in situations such as above).
- I understand that, subject to the constraints above, no recordings will be replayed in any public forum or made available to any audience other than the current researchers/research team.
- I freely and voluntarily agree to be part of this research study, though without prejudice to my legal and ethical rights.
- I understand that I may refuse to answer any question and that I may withdraw at any time without penalty.
- I understand that my participation is fully anonymous and that no personal details about me will be recorded.
- I understand that if I or anyone in my family has a history of epilepsy then I am proceeding at my own risk.
- I have received a copy of this agreement.

Student Participant Assent Form

DECLARATION:

- I assent to participate in this research. As I am under 18, I understand that permission must also be provided by my parent/guardian to participate in this study.
- I have read, or had read to me, an information form providing information about this research (as detailed in the information sheet) and this consent form.
- I understand that my participation is fully anonymous and that no personal details about me will be recorded.
- I understand that it is a staff member of [redacted] College running this study but that no information in this study will be used to identify me.
- I have had the opportunity to ask questions and all my questions have been answered to my satisfaction. I understand the description of the research that is being provided to me.
- I agree to my data being presented as part of the project work for the MSc in Technology and Learning (TCD) in a way that does not reveal his/her identity.
- I freely and voluntarily agree to be part of this research study, though without prejudice to my legal and ethical rights.
- I understand that I may refuse to answer any question and that I may withdraw at any time without penalty.
- I assent to being observed, by the researcher through note-taking, while completing the tasks associated with this project.
- I understand that in the unlikely event that illicit activities become known over the course of this research, these will be reported to appropriate authorities.
- I understand that my data will be stored securely and deleted on completion of the study.
- I understand that the study involves viewing a computer screen and that if I or anyone in my family has a history of epilepsy then I am proceeding at my own risk.
- I have received a copy of this agreement.

I ________________________consent to taking part in this research project.

Date: ___________

Signature of project leader (TCD): ____ Brendan Tangney _ Date: __04/Dec___

Statement of investigator's responsibility:
I have explained the nature and purpose of this research study, the procedures to be undertaken and any risks that may be involved. I have offered to answer any questions and fully answered such questions. I believe that the participant understands my explanation and has freely given informed consent. I undertake to act in accordance with the information supplied.

RESEARCHER: Tony Knox tonyknox@[redacted]
SUPERVISOR: Brendan Tangney tangey@tcd.ie

Board of Management
Information Sheet

TITLE OF PROJECT:
An Investigation into Bray's Heuristics for Mathematics Learning Activities as Applied to Functions

LEAD RESEARCHER: Tony Knox

BACKGROUND TO RESEARCH:
The research is part of the project work for a postgraduate degree in Technology and Learning in Trinity College Dublin, the University of Dublin, and may be included in related research publications.

The research examines the usefulness of providing students with mathematics problems that will be investigated using a website called Desmos. The research seeks to find out if students engaging with these problems with the software will lead the students to learn many properties of functions and also if their attitude to mathematics will change.

PROCEDURES OF THIS STUDY:
As part of this study participants will be asked to:
1. Complete a fully anonymised questionnaire before the study.
2. Participate in a series of twelve, 40-minute, classes during their timetabled mathematics classes, engaging with some mathematics problems using the Desmos website.
3. Complete a fully anonymised questionnaire after the study.
4. Participate in a 20-minute group interview to assess their experience of the study. I understand this interview will be recorded (audio only) and once transcribed, the audio recording will be deleted. Only the researcher will have access to this recording until the recording is destroyed.

PARTICIPATION:
The learning activities are directly relevant to the class and cover learning outcomes from the students' mathematics syllabus. All students in the class will participate in the learning activities (number 2. above), as normal. While students cannot opt out of the learning activity they can at any stage exclude themselves from the research aspects of the activity (numbers 1., 3. and 4. above).

ILLICIT ACTIVITY:
In the unlikely event that illicit activities become known over the course of this research, these will be reported to appropriate authorities.

DATA RECORDING:
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CONFLICT OF INTEREST:
Participants include students in the school where I am employed.

PUBLICATION:
The results from this study will be presented as part of the project work for a postgraduate degree in Technology and Learning in Trinity College Dublin, the University of Dublin, and may be included in related research publications.

DECLARATION:

- I am 18 years or older and am competent to provide consent.
- I have read, or had read to me, a document providing information about this research and this consent form. I have had the opportunity to ask questions and all my questions have been answered to my satisfaction and understand the description of the research that is being provided to me.
- I agree that my data is used for scientific purposes and I have no objection that my data is published in scientific publications in a way that does not reveal my identity.
- I understand that if I make illicit activities known, these will be reported to appropriate authorities.
- I understand that I may stop electronic recordings at any time, and that I may at any time, even subsequent to my participation have such recordings destroyed (except in situations such as above).
- I understand that, subject to the constraints above, no recordings will be replayed in any public forum or made available to any audience other than the current researchers/research team.
- I freely and voluntarily agree to be part of this research study, though without prejudice to my legal and ethical rights.
- I understand that I may refuse to answer any question and that I may withdraw at any time without penalty.
- I understand that my participation is fully anonymous and that no personal details about me will be recorded.
- I understand that if I or anyone in my family has a history of epilepsy then I am proceeding at my own risk.
- I have received a copy of this agreement.

Board of Management Consent Form

DECLARATION:

- I am 18 year or over and competent to provide consent.
- I have read, or had read to me, an information form providing information about this research (as detailed in the information sheet) and this consent form.
- I understand that my students' participation is fully anonymous and that no personal details about any student will be recorded.
- I understand that it is a staff member of [redacted] College running this study but that no information in this study will be used to identify any individual.
- I have had the opportunity to ask questions and all my questions have been answered to my satisfaction. I understand the description of the research that is being provided to me.
- I agree to data of students of [redacted] College being presented as part of the project work for the MSc in Technology and Learning (TCD) in a way that does not reveal their identities.
- I freely and voluntarily agree to students of [redacted] College being part of this research study, though without prejudice to their legal and ethical rights.
- I understand that participants may refuse to answer any question and that each participant may withdraw at any time without penalty.
- I consent to students being observed, by the researcher through note-taking, while completing the tasks associated with this project.
- I understand that in the unlikely event that illicit activities become known over the course of this research, these will be reported to appropriate authorities.
- I understand that students' data will be stored securely and deleted on completion of the study.
- I understand that the study involves viewing a computer screen and that if any student or anyone their family has a history of epilepsy then they are proceeding at their own risk.
- I understand that the school may withdraw from the project at any time should it wish to do so for any reason and without penalty.
- I have received a copy of this agreement.

I ________________________, on behalf of the Board of Management of [redacted] College to give consent for the research to be carried out.

Signature: ____________________________ Date: ___________

Signature of project leader (TCD): Brendan Tangney__ Date: ___04/Dec___

Statement of investigator's responsibility:
I have explained the nature and purpose of this research study, the procedures to be undertaken and any risks that may be involved. I have offered to answer any questions and fully answered such questions. I believe that the participant understands my explanation and has freely given informed consent. I undertake to act in accordance with the information supplied.

RESEARCHER: Tony Knox tonyknox@[redacted]
SUPERVISOR: Brendan Tangney tangey@tcd.ie

Appendix 5: Mathematics and Technology Attitudes Scale

Questionnaire **I.D. Number**______________

- Mathematics and Technology Attitudes Scale (MTAS) (Pierce, Stacey, & Barkatsas, 2007)
- Each question is optional. Feel free to omit a response to any question; however the researcher would be grateful if all questions are responded to.
- In the extremely unlikely event that illicit activity is reported I will be obliged to report it to appropriate authorities.

No.	Statement	Strongly Disagree	Disagree	Not Sure	Agree	Strongly Agree
1	I concentrate hard in mathematics					
2	I try to answer questions the teacher asks					
3	If I make mistakes, I work until I have corrected them					
4	If I can't do a problem, I keep trying different ideas					
5	I am good at using computers					
6	I am good at using devices like games consoles, iPods, smartphones etc.					
7	I can fix a lot of computer problems					
8	I can master any computer programs or apps needed for school					
9	I have a mathematical mind					
10	I can get good results in mathematics					
11	I know I can handle difficulties in mathematics					
12	I am confident with mathematics					
13	I am interested to learn new things in mathematics					
14	In mathematics you get rewards for effort					
15	Learning mathematics is enjoyable					
16	I get a sense of satisfaction when I solve mathematics problems					
17	I like using Technology for learning mathematics					
18	Using Technology tools in mathematics is worth the extra effort					
19	Mathematics is more interesting when using Technology tools					
20	Technology tools help me learn mathematics better					

Appendix 6: Group Interview Protocol and Questions

Group Interview Protocol and Questions

Thank you for participating in this group interview. The interview will be recorded so that I can type out the transcript of the interview. I will not include any of your names in the transcript. No one other than me will hear the audio file and the audio file will be deleted once the transcript is complete. Your names will not be used in the transcript to preserve your anonymity. Excerpts of the transcripts will be used within the dissertation I will be submitting to Trinity College Dublin in partial fulfilment of the requirements for the degree of Master of Science in Technology & Learning. This will be the only place these excerpts will be published.

Just to remind you your participation is voluntary and you can change mind about it at any time. Each question in this interview is optional and you are under no obligation to answer any question. If you decide to opt out during the interview I will not use any of the information that you have discussed. If you have any questions at any time, please don't hesitate in asking me at any stage in the interview.
Please do not name third parties in any answers. Any such replies will be anonymised.
In the extremely unlikely event that illicit activity is reported I will be obliged to report it to appropriate authorities.

I want you to be totally honest when answering the questions and not to say what you think I might want to hear, but say what you really think. There are no right or wrong answers. This interview is about your opinion.
If everybody is comfortable we'll begin.

1. Can you describe your usual maths class?

2. Can you tell me what you think I was trying to achieve?
What was different to your usual class?
How did you feel about it? Why?
Has the experience changed how you feel about maths?
Are you more curious about where/when/how maths is used?
Would more experiences like this have a positive/negative impact on your relationship with maths? Why?

3. What did you learn?
Did you learn any new mathematical content that you had not seen in class?
Has this changed your confidence in your ability to understand or use the maths?
Do you think you gained any new skills – mathematical or otherwise?

4. Did you find the tasks challenging?
Did you find the tasks interesting?

5. In what ways did you find working with the computer unhelpful?
In what ways did you find working with computer helpful?

6. In what ways did you find working as part of a group helpful?
In what ways did you find working as part of a group unhelpful?

7. What is your opinion on the group presentations? Did you learn anything from the group presentations?

8. What did you like/dislike about the experience? Would you suggest doing anything differently?

9. Do you want to add anything? Is there anything I haven't asked that you want to add about the experience?

10. In one word, how would you sum up the experience?

Appendix 7: Open Coding Sample

Transcript of Group Interview 1

TK: Can you describe your usual maths class?

1: Well, em, it mainly of consists of just taking stuff down off the board and doing questions that you give us.

TK: Would you agree with that? Would you expand on that here?

2: Yeah, we just do kind of do new stuff and make sure we know how to do like certain things and get better at it. *class learn math*

TK: And how about yourself? Would you like to expand on that there? Can you describe your usual maths class and again be honest

classboring
3: Well sometimes it can be a bit boring, because it's just, I don't know.... I don't know, it's kind of my own opinion because I just don't really like maths in general but, yeah, it's kind of what they said it's just learning new things and *maths neg. pre.*
~~learning pre~~ class learn math

TK: And how you learn them is it similar to what X was saying?

4: We take it down off the board and then you kind of come round to everyone individually to help them in different ways kind of.

TK: Ok. That's grand, perfect, lovely, can you tell me what you think I was trying to achieve with the maths classes that were in the computer room? Ok, so, do you want to start?

different *use* *tech*
4: Trying to like show an alternative way of, like, using maths and incorporate technology into like an educational way.
learning

TK: Brilliant, lovely, yeah. How about here?

different
3: I would've said the same. A different way learning.

classboring *engage*
2: Well like, it's not as, you know boring as looking at the board and writing it down. It's more we interact a bit more.
Group pos. *Engage*

[General agreement] *Group pos. x3* *Engage x3*

learn math *Tech. pos.*
It makes more sense to us when it comes from the technology.

TK: Ok. And yourself?

Group pos *Talk* *Group People Friends*
1: The group work is just a lot more interactive and you talk to your friends in your group and you get to work with people you wouldn't normally talk to.
work *people group*

TK: How did you feel about the lessons in the computer room?

Transcript of Group Interview 1 1

Appendix 8: Frequency Table from the Open Coding Process

	Normal Class		
1	Class boring	Normal class boring or uninteresting	5
2	Class normal	Normal class is normal	4
3	Class pos.	Thoughts or feelings about normal class are positive	10
4	Class enjoy	The normal class is enjoyable	1
5	Class neutral	Thoughts or feelings about normal class are neutral	1
6	Class teacher work	Teacher in normal class works/says a lot	2
7	Class learn math	Maths is learned in the normal class	9
8	Class group learn	Learning benefits from group work in normal class	7
	Attitudes		
9	Maths neg. pre.	Negative feelings towards maths before learning experience	2
10	Maths pos. pre	Positive feelings about maths prior to the learning experience	4
11	Confidence, no change	No change in confidence due to learning experience	3
12	Attitude increase	Increase in positive thoughts or feelings towards maths	15
13	Behaviour increase	Concentrating more	1
14	Attitude, no change	No change in attitude (unspecified) due to learning experience	13
15	Maths not useful	Maths not useful in the real world	1
16	New view on maths		1
17	Maths, no change viewpoint	"It hasn't changed my view on maths"	2
18	Curious	More curious about maths as a result of learning experience	2
	Experience as a Whole		
19	Pos. Exp.	A positive experience	32
20	Neg. Exp.	A negative experience	1
21	Neutral Exp.	Feelings about the experience were neutral e.g. "it was alright".	1
22	Enjoy	Experience was enjoyable, fun or entertaining	8
23	Creative	Creativity occurred	2
24	Challenging	Aspects of the learning experience were challenging	31
25	Manageable	Aspects of the learning experience were manageable. Challenges were overcome	24
26	Little challenge	Found little challenge in the maths of the experience	1
	Divergent Tasks		
27	Divergent Task Pos.	Positive view about the divergent thinking tasks	3
	Engagement/Variety/Repetition/Different from Norm/Repetitive/Boring/Novelty		
28	Novelty	Positive thoughts and feelings about experience stronger at the start	6
29	Variety	The activities were varied	2
30	Boring	Aspects of the learning experience were boring	9
31	Repetitive	Thoughts on the activities in the experience being repetitive and needing more variation.	14
32	Engage	Learning experience was engaging	29
33	Different	Experience was different to the normal class	21
	Learning (Note: two large areas of learning in "Group" section too)		
34	Learn math	Mathematics was learned	23
35	Little maths learning	Only learned a little maths in the experience	2
36	Learn math none	No maths was learned	1
37	Learning	Learning (in an unspecified topic) occurred	2

	Groups		
38	Group	Groups mentioned in an unspecified way	4
39	Group pos.	An aspect of working as part of a team was positive (unspecified whether it was for learning)	11
40	Group neg.	An aspect of working as part of a team was negative (unspecified whether it was for learning)	1
41	Group learn	Learning benefitted from group work, it was easier to learn in a group, sharing of ideas to solve problems	27
42	Learn group	Learned about group work, improved in group work skills.	9
43	Group labour	Thoughts on how the work was not done by all of the team at times	12
	Technology		
44	Tech. Pos.	Technology had some benefit	32
45	Tech. Neg.	Technology was a disadvantage	5
46	Tech. Neg. Paper	Moving between technology and paper (if required) was awkward	1
47	Tech. Important	Belief that technology is important to learn about	1
48	Learn Tech.	Learned how to use technology	2
49	Tech.	Technology mentioned in an unspecified way	4
50	Manipulate	Manipulating and changing something on the screen	2
51	Technical Issues	Computers freezing	3
	Presentations		
52	Pres. neg. attitude	Negative attitude to the idea of a group presentation (at the start)	1
53	Pres pos.	Positive feelings or thoughts about the group presentations	5
54	Pres. neg.	Negative feelings or thoughts about the group presentations	1
55	Pres. neutral	Neutral feelings or thoughts about the group presentations	3
56	Pres. variety	Variety as a theme mentioned in relation to the group presentations	6
57	Pres. interesting	Group Presentations were interesting	1
58	Pres. challenging	Some aspect of the group presentation was challenging e.g. putting it together	4
59	Pres. no learning	No learning from group presentations	1
60	Pres. learn math	Maths was learned from the group presentations	11
61	Pres. nervous	Nervous about presenting	1
	Format/Group size/Group participants/Length/Spread		
62	Format change	Changing format from in computer room for consecutive classes to having some classes in a normal classroom in between	7
63	Pairs/Threes	Suggestion to work in pairs or in a group of three	4
64	Intervention spread	Intervention spread over a long time. Hard to remember first activities	1
65	Long	Not have the course as long as it was	2
66	Group pick own one	Better if participants could pick their own teams	1
67	Change groups	Asked to change the teams round	2
	Uncategorised		
68	More	Expression that there could have been more to the learning experience.	3
69	Use	Using maths	1
70	Talk		1
71	People		3
72	Friends		1
73	Work		1
74	Different learner needs	Different people learn in different ways e.g. writing, books, computers	6
75	Graphs	Mention of graphs	4
76	Books		2
77	Books, neg.	Negative thoughts about using books	3