

# Application of Splines and Wavelets Smoothing Techniques For Functional Data

Mohammad Mahdi Eslami, Master of Science in Computer Science University  
of Dublin, Trinity College, 2022

Advancement in statistical technologies has empowered scientists to collect more sophisticated data. This increasing sophistication demands a new conceptualization and consequently, new set of tools and techniques to extract additional information from the underlying patterns of the data. Functional Data Analysis is the study of examining the variability of a dataset when the observations are curves instead of discrete data samples, enabling statistical tools to further study the derivatives of the curve which may contain relevant information to the subject. The first step in Functional Data Analysis is to create smooth approximation functional curves from a set of discrete data points for which some approximation methods are required to describe the behavior of the data. Among the smoothing techniques to create the approximation function in Functional Data Analysis, Splines and Wavelets are of interest in this paper. Splines in nutshell employ a set of knots and fit piecewise polynomial functions between them to provide an estimation function for the data. Wavelets apply smoothing by decomposing the data into set of frequency sub-bands and filter out data samples which do not lie in the specified range. This will remove rapid changes from the data pattern, resulting in a smooth estimation.

The main purpose of this paper is to study the fundamental concepts involved in Splines including different families of splines and different knot placement strategies. Multiple wavelet basis as well as the viability of each choice is also discussed in this paper. The smoothing factors employed by each method to create an estimation function are also studied in this paper by applying the methods different timeseries and spectrogramic datasets. This study also provides a practical guide through which these techniques can be applied on functional datasets. An introduction and evaluation of various related packages available both in R and Python is included in the practical guide. We implement Splines in R and use PyWavelet library for applying wavelets. Both methods showed competency to be utilized as a smoothing technique for functional data.

Keywords: Functional Data Analysis, Smoothing, Curve-fitting, Splines, Wavelets, Spline Knot Placement, R, Pywavelet.