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# A Fast Method Solving Integral Equation-Based Eletromagnetic Propagation Model

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of the requirements for the degree of

Master of Science in Computer Science (Data Science)

# Declaration

I, the undersigned, declare that this work has not previously been submitted as an exercise for a degree at this, or any other University, and that unless otherwise stated, is my own work.

First Last

Month Day, Year

## Acknowledgement

I would like to start my acknowledgement by expressing my gratitude for to my supervisor Professor Dr. Éamonn Ó Nualláin, not only for suggesting the topic for my dissertation but also for all his help during the process.

I would like to thank Tom Quinn who is my friend helped me a lot on everything. Special thanks for Dr. Zhengnan Cao and Dr. Ke Ren who gave me suggestions while I was preparing to write the report during the last few months.

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# Abstract

A fast method for electric field integral equation to evaluate the signal attenuation at 970MHz is described here. An 700-meter, two-dimensional terrain profile is used with the approximation of perfectly electrically conducting (PEC) segments. The proposed algorithms are not working as expected. The detailed steps of implementing EFIE method is descried as well as other fast method of solving EFIE. The reasons of divergence are discussed.

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# 1. Introduction

## 1.1 Motivation

With the increasing number of all kinds of applications on mobile phones, the requirement for wireless communication is also increasing dramatically. How to plan the antennas with transmitters to coverage area as much as possible with good quality signal strength is the problem that every system designer needs to solve. Electric Field Integral Equation(EFIE) calculates electric field strength with the terrain profile. The project is motivated by the paper[14] written by Professor Dr. Éamonn Ó Nualláin. The paper proposed a fast method that leads to an accurate result of electric field strength. This method significantly reduces the time of calculating the electric field strength over an area. A fast method using iterative methods is proposed in the project.

## 1.2 Project Goal

The goal of the project is to provide a fast method that will implement iterative methods Jacobi method, and the Gauss-Seidel method to solve the Electric Field Integral Equation (EFIE) to reduce the running time of the program used to solve the EFIE. Few other goals need to be achieved to support the knowledge of the topic area.

1. Research state of the art in the area of wireless communication.
2. Research the models for predicting the attenuation of the signal.
3. Research the statistics characteristics of the signal fading.
4. Research on the Cognitive Radio and Radio environment map.

# 2. State of Arts

In the chapter, we will introduce some core conceptions about the attenuation characteristics of radio transmission, such as path loss, shadowing, and fading. We will also discuss the models used to help the communication system designers predict the signal strength with the given distance or the transmitter's coverage. Section 1 includes some basic ideas about the composition of a communication system. In section 2, we will go through the different propagation characteristics of the communication system along with the models and statistic tools used to predict the attenuation. In section 6, A multi-dominate database REM, which stands for Radio Environment Map, is introduced as well as how the REM can be used to improve the usage of the spectrum and other benefits. Also, some unsolved problem related to REM is discussed. Last but not least, the Electrical Field Integral Equation (EFIE) is introduced and its drawbacks.

## 2.1 Wireless Communication

A Wireless Communication system is an information transmission system without using any electric conductors, like wires, or cable, when it propagates from the source to the destination in the form of electromagnetic waves. Some fundamental conceptions used in a wireless communication system are introduced.

### 2.1.1 Wireless Channel

Wireless Channel is the critical concept for understanding how we could design a wireless system and operate a wireless system and also how we could do an analysis of a wireless system. In Claude Shannon's paper [1], A generic architecture communication system is introduced. An information source tries to send the message to a destination. The message is converted to a signal that suits the transmitter, which will send the converted signal through the channel. The channel will modify the signal unpredictably sent by the transmitter. And then, the receiver at the end of the destination receives the converted and modified signal. The receiver is designed to resort signal and then deliver the message to the destination so there will be fewer errors in the message.

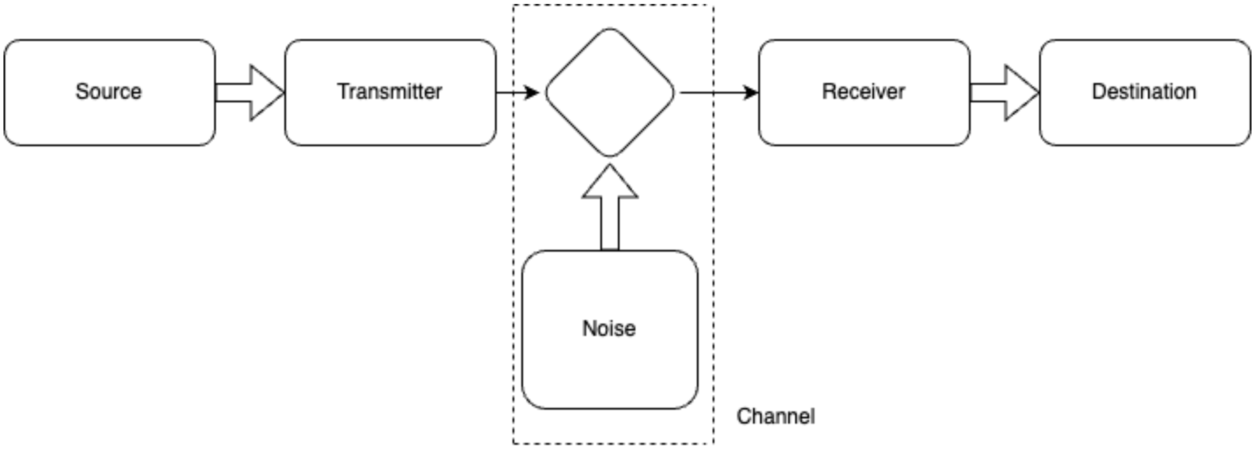


Figure 2.1 Generic Architecture communication system, source from [3]

From Figure 2.1, the diagram between transmitter and receiver is treated as the channel.

### 2.1.2 Electromagnetic Spectrum

Spectrum is the radio source, the basic resource used by the wireless communication system. In real practice, the frequency range of the Spectrum is from $3kHz$ up to $300GHz$, with the corresponding wavelength from $100km$ to $1mm$, respectively, using the equation followed by

$$\lambda = \frac{c}{f}$$

where the c is the wave speed with a given medium, such as air, we can treat it as light speed in a vacuum with the approximate value $3 \times 10^8\ m/s$.

The electromagnetic spectrum can be divided into different frequency bands [3].

| VLF | LF | MF | HF | VHF | UHF | SHF | EHF |
|---|---|---|---|---|---|---|---|
| 3-30kHz | 30kHz – 300 kHz | 0.3MHz – 3MHz | 3MHz-30MHz | 30MHz-300MHz | 0.3GHz-3GHz | 3GHz-30GHz | 30GHz-300GHz |

Different bands of the spectrum can have different uses. For example, SHF can be used in radar, and UHF can be used to propagate the signal to the television. In this paper, we will only consider VHF and UHF bands.

### 2.1.3 Noise Source of the Channel

From Figure 2.1, which can be used to represent Shannon's generic architecture of wireless communication systems, we can see that the noise in the wireless channel is the primary source of the interference that modifies the signals during the transmission. The noise source in the wireless environment can be divided into two parts[3], *Multiplicative* noise and *Additive* noise. The Additive noise is defined as those caused by 1) receiver generating within itself. 2) external sources, for example, the atmospheric effect, cosmic radiation, and other interference from other electrical appliances. 3) The reused channel is applied to maximize the usage of the systems. Multiplicative noise is encountered in how the electromagnetic waves are transmitted from the transmitter to the receiver. The noises can be generated as the following: 1) The *directional characteristics* of the transmitter and receiver. 2) the *reflection* from the smooth surfaces like hills. 3) *Absorption* is caused by clutters like trees and the atmosphere between the transmitter and receiver. 4) *Scattering* arises from the rough surface, rough ground, like the sea surface. 5) Diffraction encountered on the edge of the obstruction during the transmission and 6) *Refraction* that the different materials of the medium.

In the last section, we have introduced how multiplicative noise is generated. In summary, multiplicative noise can be divided into three main types of fading. They are 1) *Path Loss*, 2) *Shadowing*, and 3) *Fast fading* (mainly small-scale fading), and we will talk about each of these three types of fading in the rest of the sections. The total signal can be treated as the superposition of Path Loss, Shadowing (large-scale fading), and Fast fading (small-scale fading, small-scale fading is not equal to fast fading).

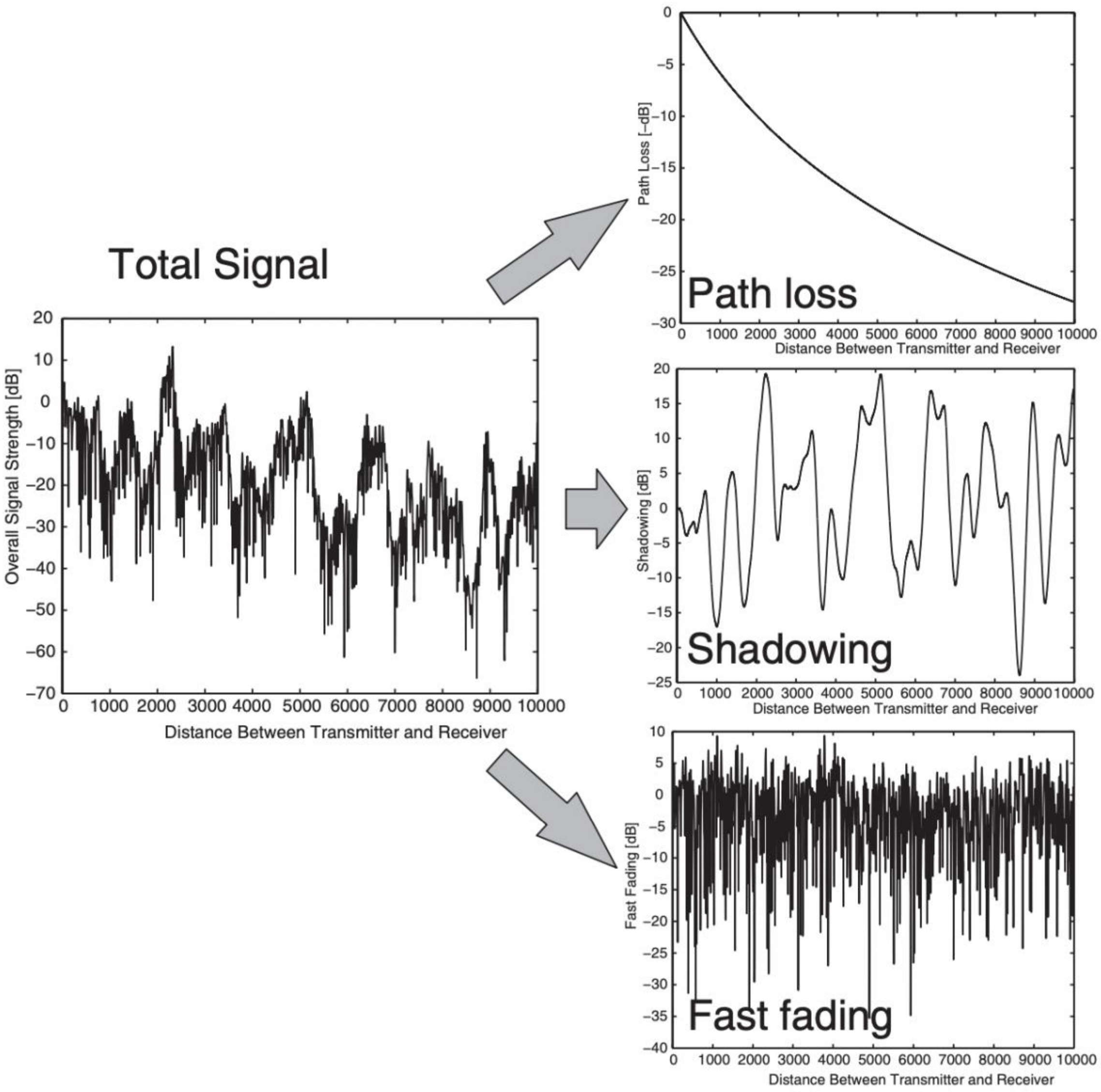


Figure 2.2, Superposition of the signal source from [3]

## 2. Path Loss

In this section, we will introduce some basic propagation models to calculate the range of wireless communication systems, considering path loss only. We can use them as an approximate model to estimate the path loss in the real world.

***Path Loss*** is the power loss between two antennas when the signal is transmitted from transmitter to receiver. It is defined as the ratio of the power of transmitted from the transmitter to the power of receiver receives as equation (2.2).

$$Path\ Loss, L = \frac{P_T G_T G_R}{P_R L_T L_R}$$

Where the $P_T$ is the transmitter's power, $G_T$ is the power gain from the transmitter side. $G_R$ is the power gained from the receiver. $P_R$ is the power received by the receiver. $L_T$ and $L_R$ are the internal loss of power.

In the following sections, we will introduce propagation models to help calculate the radio system's maximum range without considering the other two fadings. By using these models, the system designer can have a good overview of the signal coverage of the transmitter.

### 2.2.1 Free Space Path Loss

The Free Space Path Loss model is the basic model in the condition that there are no obstacles between the transmitter and receiver. It is the simplest model, and equation (2.2.1) is the free space loss in decibels[3]

$$L_F = 32.4 + 20\ logR + 20\ logf_{MHz}$$

Where R is the distance from the transmitter antenna to the receiver in kilometers, $f_{MHz}$ is the frequency of the signal in megahertz. The Eq(2.2.1.1) is the Friis transmission formula[2] in the decibel description. The Friis transmission formula is shown below:

$$\frac{P_t}{P_r} = \left(\frac{4\pi r}{\lambda}\right)^2$$

Where the ratio of $P_r$ and $P_t$ is the path loss definition, $r$ is the distance between antennas. $\lambda$ is the signal's wavelength, also can be written as $\frac{c}{f}$. Then the formula can be rewritten as

$$\frac{P_t}{P_r} = \left(\frac{4\pi r f}{c}\right)^2$$

The formula is suggested the path loss in the free space only.

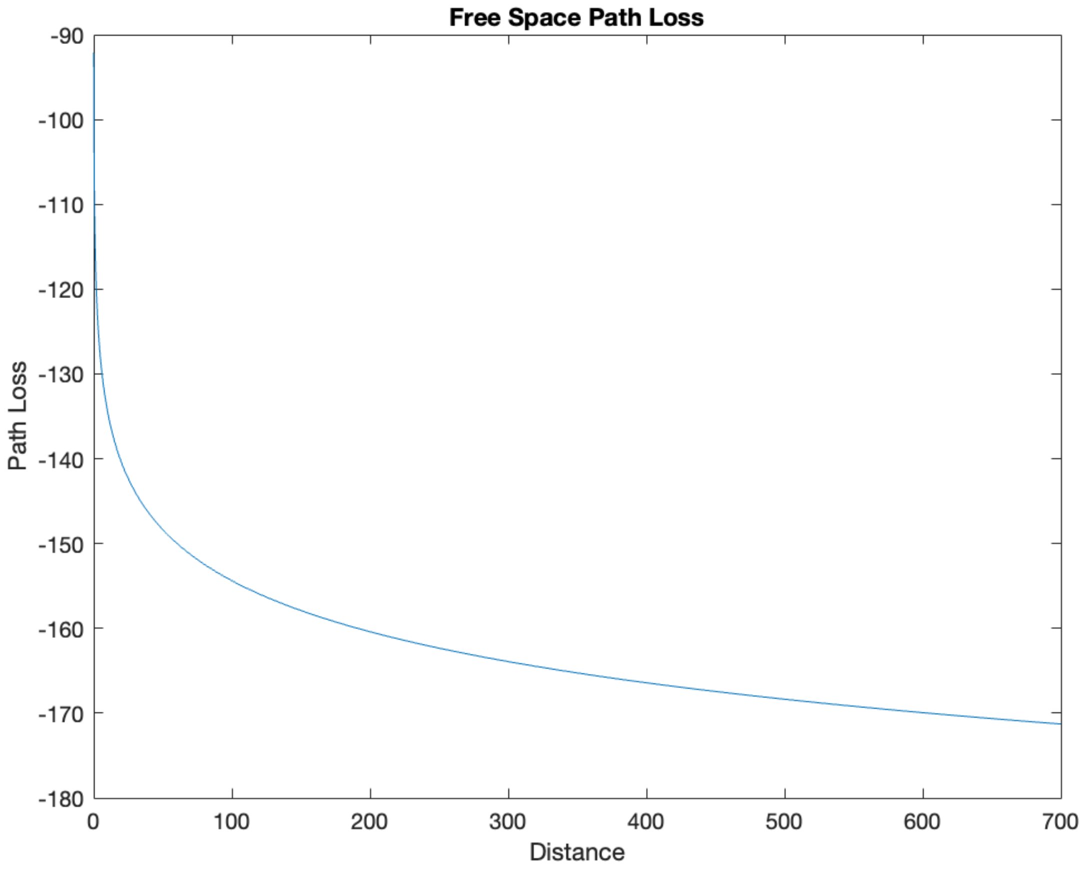


Figure 2.1 Free Space Path Loss Model, Path Loss[dB] vs Distance[km]

Figure 2.1 shows the Path Loss versus Distance using the Eq (2.2.1.1) with a frequency value of 970MHz. The X-axis represents the distance R in kilometers while thy y-axis represents the power loss but takes the negative values to show the loss of power. This figure shows that the path loss increases with the distance increase, and the curve's trend follows the logarithm tendency.

The Free Space Path Loss is calculated as the minimum loss for the given distance in the practical consideration, which means that the path loss during the transmission between two antennas (transmitter and receiver), the minimum power loss is calculated. Therefore, the maximum range of the transmitter can provide coverage, in theory, can be discovered.

### 2.2.2 Plane Earth Loss

In this section, we will introduce another basic propagation model to calculate the path loss called ***Plane Earth Loss Model***. This model considers a more practical transmission situation that considers the ground reflection of electromagnetic waves. The Eq(2.2.2.1) is the Plane Earth Loss Model in the decibels form.

$$L_{PEL} = 40\ logr - 20\ logh_m - 20\ logh_b$$

Where $r$ is the distance between the base station and mobile station, $h_m$ is the height of the mobile station, and $h_b$ is the height of the base station. However, the equation is only the approximation-based assumption. We can see that Eq (2.2.2.1) is independent of frequency.

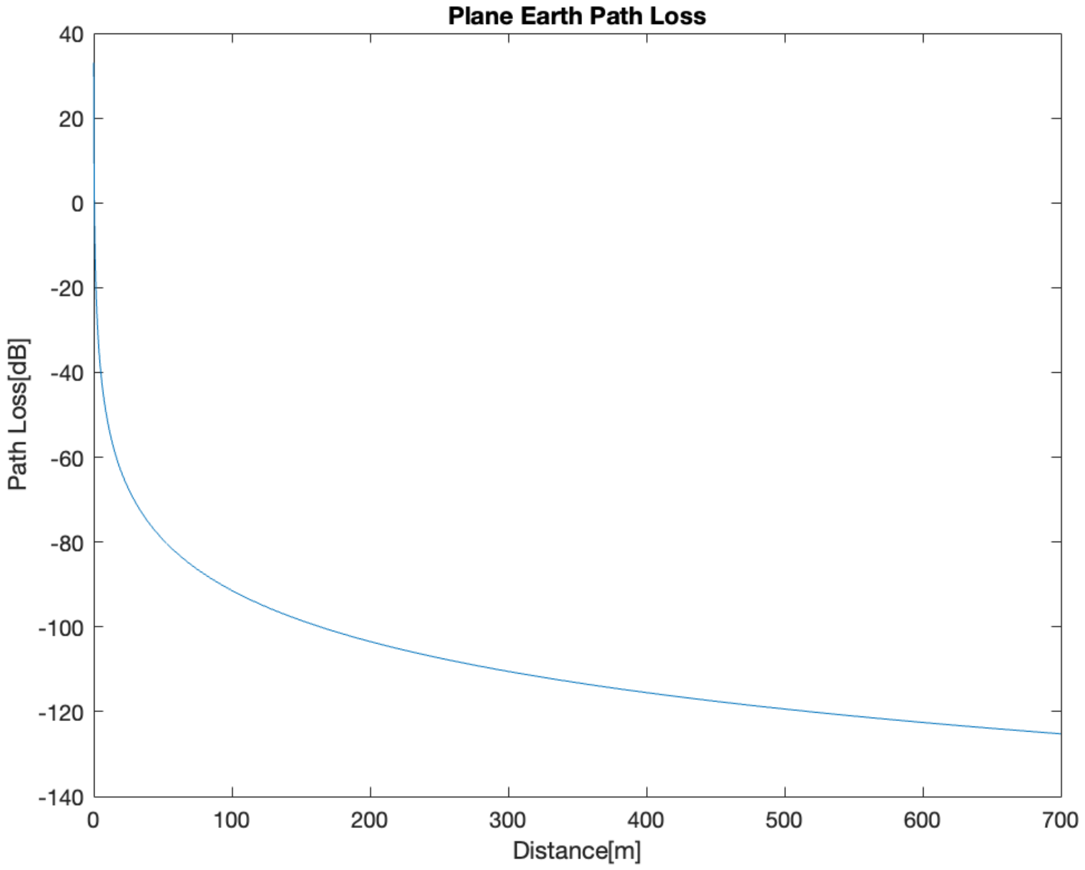


Figure 2.2 Plane Earth Path Loss Model, Path Loss[dB] vs Distance[m]

Figure 2.2 shows the path loss versus distance in the meter that uses the Plane Earth model to predict. The base station height is 30m, and the mobile station is 1.5m, with the same frequency at 970MHz. Then we take the negative values to represent the loss of power. The power loss increases with the distance increases. It has the same tendency as Free Path Loss Model.

The assumption needs to be mentioned for Eq(2.2.2.1). The assumption is that the height of both base station and mobile antennas are small compared to $r$ , the distance between the base station and mobile.

$$h_m,\ h_b << r$$

With this assumption, we can get the simplified path difference between the reflection path and the direct path, which is

$$(r_2 - r_1) \approx \frac{2h_m h_b}{r}$$

The Eq(2.2.2.2) is the difference between the reflection and direct paths calculated using image theory. The image theory indicates that we can treat the reflected ray from the other transmitter in the mirror symmetry position of the actual transmitter. As shown below

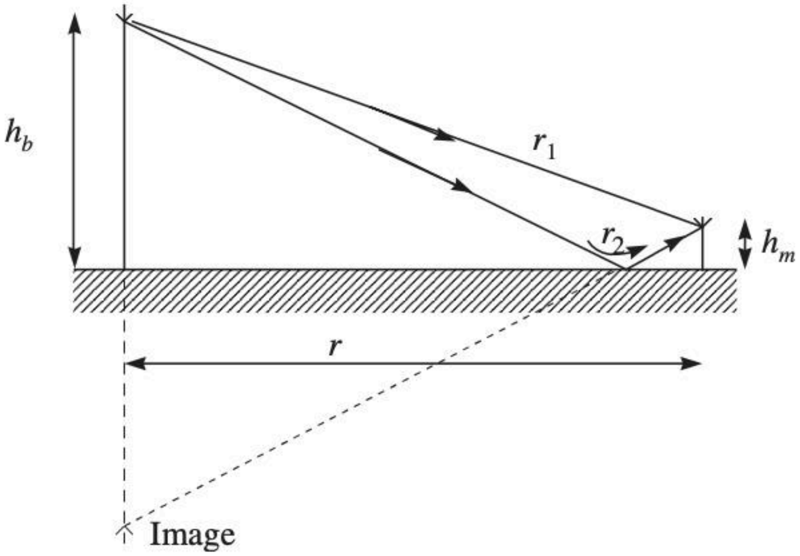


Figure 2.3 Image theory diagram, source from [3]

Therefore the different path length of the reflection path and direct path can be described as follow.

$$r_2 - r_1 = r\left[\sqrt{\left(\frac{h_b + h_m}{r}\right)^2 + 1} - \sqrt{\left(\frac{h_b - h_m}{r}\right)^2 + 1}\right]$$

Moreover, due to this assumption, we can assume that the amplitudes of the electromagnetic wave can be treated as identical[3].

The ***Plane Earth Path Loss*** model is not accurate in practical. The model is only applicable to long distances as the assumption mentioned above.

### 2.2.3 Okumura-Hata Model

Okumura-Hata Model is the empirical model used to predict propagation loss in a wide area. The complexity of the radio environment in a wide area can be high, and the models we mentioned in the previous section are impossible to handle. In the typical situation, we use Terrestrial Fixed Links propagation prediction when calculating the propagation loss between two antennas. In this method, we need the path profile, which contains the details of the terrain profile between two antennas. However, the amount of data in the terrain profile can be enormous in wide areas, so that the computation efforts can be very high.

Therefore, for those engineers who need to predict that path loss over the area, an empirical method is more suitable to use. That is why we introduce the empirical model, and the ***Okumura-Hata Model*** is the most widely-used and accepted empirical model used in a wide area today.

The simplest empirical path loss model is described as follows in decibels form

$$L = 10n\ log\ r + K$$

where $r$ is the distance between the base station and the mobile. Where $K = -10log_{10}k$, k can be considered the propagation loss in 1 meter. Moreover, n is the path loss exponent, which depends on the communication system environment parameters such as the base station's height. Free Space Path Loss and Plane Earth Path Loss can be described in this way.

The Okumura-Hata model was introduced by Okumura[4] firstly, and Hata[5] made a series of approximations that made the model more computable. The model is based on the data measured in Tokyo, with the frequency range between $150MHz$ to $1500MHz$. To use the Okumura-Hata model to predict a large wide area, we need to consider the type of area we want to predict. In both papers, Okumura[4] divide the area into three different categories: *Urban Area*, *Suburban Area*, and *Open Area*. The table below has detailed the definition of the different areas:

| Area Categories | Definition |
|---|---|
| Open Area | The area that has no tall trees, tall buildings in the path of transmission, such as farmland |
| Suburban Area | Villages or motorways with scattered buildings like trees or houses will have obstacles between the paths. |
| Urban Area | Cities or towns with many buildings, houses, or large villages with tall trees. Also, the size of the cities is a factor taken into account. The large city means those cities where the average building height is above 15 m while the other cities where the average building height is below the 15m are called small or medium cities. |

The mathematics expression for different categories of the area is different. The formulas following is the predictions of path loss calculated using Hata[5] approximations:

For Urban Areas:

$$L_{dB} = A + B\ logR - E$$

For Suburban Areas:

$$L_{dB} = A + B\ logR - C$$

For Open Areas:

$$L_{dB} = A + B\ logR - D$$

Where

$$A = 69.55 + 26.16 \cdot log f_c - 13.82 \cdot log h_b$$

$$B = 44.9 - 6.55 \cdot log h_b$$

$$C = 2\left(log\left(\frac{f_c}{28}\right)\right)^2 + 5.4$$

$$D = 4.78(log f_c)^2 - 18.33 log f_c + 40.94$$

Where $f_c$ and $h_b$ are the base station's frequency and height, respectively, there is a validation range of these two parameters, which is that $150MHz \leq fc \leq 1500MHz$ while $30m \leq h_b \leq 200m$. R is the distance of the path, which needs to be not less than 1km and up to 20km to be valid to use the formula, and $h_m$ is the height of the mobile station, which the valid range needs to be within $1 - 10\ m$.

For the *Urban Area*, since there are two other subdivisions of small and medium cities and large cities, $E$ has a different equation:

For medium to small cities

$$E = (1.1 \cdot log f_c - 0.7)h_m - (1.56 \cdot log f_c - 0.8)$$

While the radio environment of the large cities is more complex than that of the medium and small cities, $E$ for large cities has another two equations for different ranges of the frequency value. The formula shows as follows:

For the frequency range from $150MHz \leq f_c \leq 200MHz$

$$E = 3.2\big(log(11.75h_m)\big)^2 - 4.97$$

For the frequency range from $400MHz \leq f_c \leq 1500MHz$

$$E = 8.29\big(log(1.54h_m)\big)^2 - 1.1$$

Figure 2.4 shows the results of different city types with frequency $970MHz$, base station height $h_b = 50m$, and the height of mobile station $h_m = 1.5m$. The values take as negative to show power loss. When calculating the path loss using the Okumura-Hata model, we set the R range as $1km - 20km$.

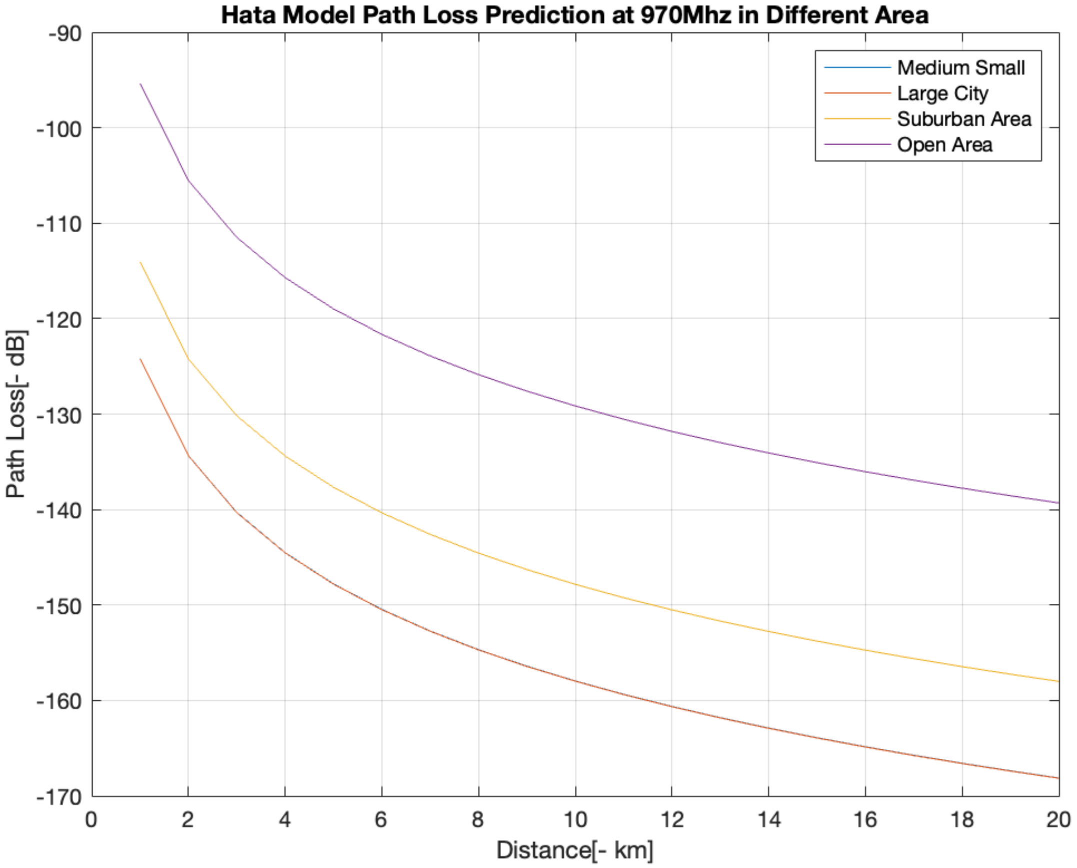


Figure 2.4 Hata-Model Path Loss [dB] vs Distance[km]

From the graph, we can see that the path loss increases when the distance increases in general. Surprisingly, the result shows that the path loss of a Large city is very close to that of a Medium Small city (the line (blue line) of the medium-small city that indicates the path loss is blocked by the line (Orange) of a large city, the line at the bottom). Also, we can see that the path loss increases with urbanization. The very top purple line in the figure is the line that shows the path loss in the *Open Area*. Compared to the other three lines, Open Area has less path loss while *Suburban Area*, the yellow line, takes the second place in path loss. In the Suburban area, the path loss is greater than the path loss in Open Area, but less path loss when compared to the Urban area, no matter the size of the cities.

As mentioned before, the Okumura-Hata model is based on the data measured from Tokyo city, and the other cities in the rest of the world have different characteristics. That may cause a small error in prediction if using the model to predict the path loss in other cities. However, the Okumura-Hata model is easy to use when designing a real-world system and is used as the standard model to measure and compare new models. This model is widely accepted and used in some commercial tools to make predictions [3].

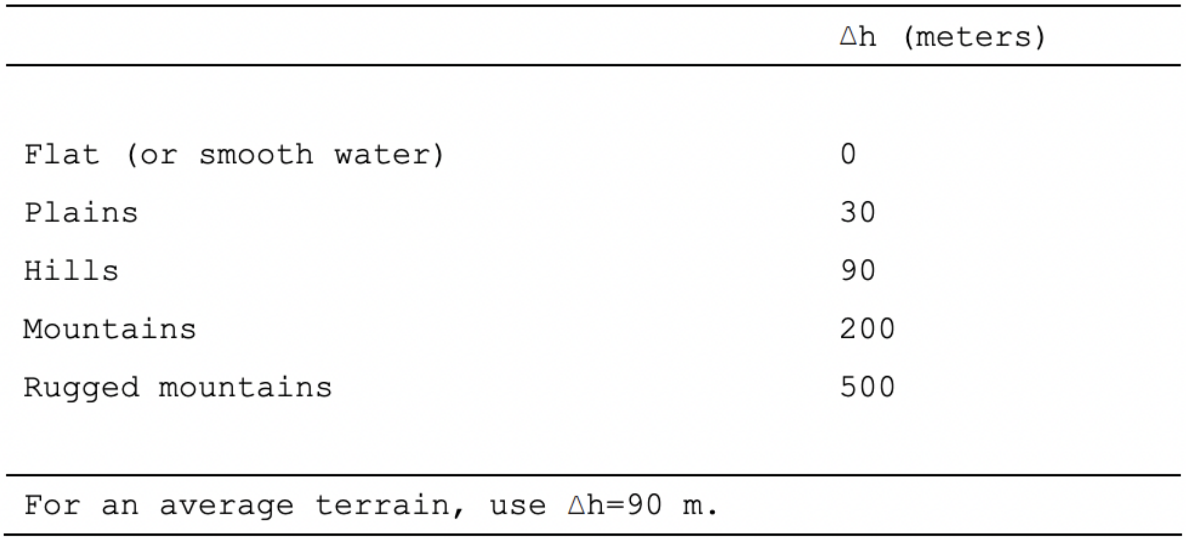
### 2.2.4 Longley-Rice Model

Another model called the Longley-Rice Model is proposed to predict the path loss in a more complex terrain profile and environment parameters. The model is known as the Irregular terrain model(ITM). The Longley-Rice model was published in the report[10] in 1968, including the computer program. The frequency range starts from 20 $MHz$ to 20 $GHz$ and extends from 1 $km$ to 2000 $km$. And the Antennas height ranges from 0.5$m$ to 3000$m$ .

The Longley-Rice model does not require detailed characteristics of the environment in the channel. Instead, it uses statistical estimation to describe the overall attenuation of the signal. Therefore other parameters[13] related to the environment need to be provided based on the actual situation

1. Terrain irregularity parameter $\Delta h$ is the full range of height above the sea level after removing the first 10% lowest and lastest 10% highest height.

   Figure 2.5 below shows a table of the different parameters used in different situations.

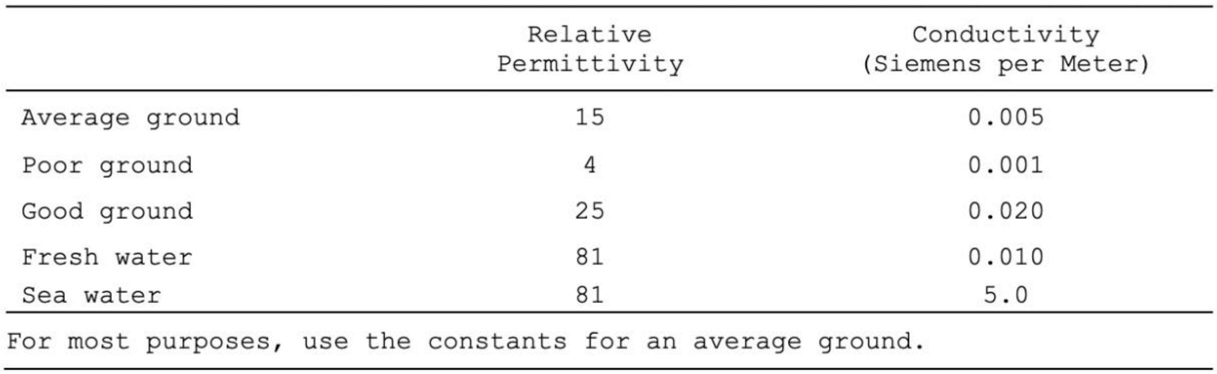


|  | Δh (meters) |
|---|---|
| Flat (or smooth water) | 0 |
| Plains | 30 |
| Hills | 90 |
| Mountains | 200 |
| Rugged mountains | 500 |
| For an average terrain, use Δh=90 m. |  |

Figure 2.5 Suggested values for Terrain Irregularity Parameter, source [13]

1. The electrical ground constants which is the relative permittivity and the conductivity of the ground

   Figure 2.6 shows the table from the report [13] suggesting the value that should be taken based on different situations.



| | Relative Permittivity | Conductivity (Siemens per Meter) |
|---|---|---|
| Average ground | 15 | 0.005 |
| Poor ground | 4 | 0.001 |
| Good ground | 25 | 0.020 |
| Fresh water | 81 | 0.010 |
| Sea water | 81 | 5.0 |

For most purposes, use the constants for an average ground.

Figure 2.6 Suggested values for Electrical Ground Constants, source [13]

2. The surface refractivity, which is the constant of the characteristics of the atmosphere
3. The climate has a close relation with surface refractivity.

   Figure 2.7 shows the table from the report[13] representing the composition of surface refractivity and climate.

Figure 2.7 Suggested values for climate and surface refractivity, source [13]

There are still other parameters that we need to choose if we want to use the Longley-Rice model, but we will not be going to talk about more details.

### 2.2.5 Summary

Path loss is an important parameter to ensure that the receiver can receive the signal sent by the transmitter. When the power of the transmitter and the receiver's sensitivity is generally settled, the path loss can be used to calculate how far away a receiver can be set up. So far, we have gone through three easy-implement but important path loss models to help us calculate the path loss: *Free Space Path Loss*, *Plane Earth Path Loss*, *Okumura-Hata Model*, and Longley-Rice Model. The free space path loss model is the basic model used as the minimum prediction path loss model, and also, it is used in the other three models we mentioned. The free space path loss model is the path loss between transmitter and receiver with no obstacles.

In comparison, the *Plane Earth Path Loss* takes the ground reflection wave into account with the direct wave. The reflection on the ground can cause more power loss, but it is independent of frequency. The *Okumura-Hata model* is an empirical model used to calculate the path loss in a large area. It takes more details into the formula and has a different correction on the different types of the areas, such as Urban Area, Suburban Area, and Open Area. Urban Area can be split into a large city and medium_small city, and for the formulas of a large city, the different frequencies can have different corrections on the formula. Nevertheless, in the real world, due to the difference in the path of propagation, the path loss can be different with the number of obstacles along the path within the given range. Longley-Rice model describes a more complex environment (such as different topography and climates) attenuation of the signal by applying statistical estimation.

## 2.3 Large-Scale Fading

In the last section, we discussed path loss during the transmission. When we talk about path loss, it is treated as the environment alongside the different paths being the same, but in the real world, that is not correct. Each path may have different sizes of buildings and different numbers of trees from other paths of the signal transmission area so that the power loss caused by extra clutter can vary. If fewer obstacles are alongside the path, the receiver will receive a more robust signal. In contrast, if there are more obstacles, the power that the signal carries to the arrived receiver will be weaker. The process is called *Large-Scale Fading* or *Shadowing*.

The book[3] Chapter 9.2 states an experiment measuring the signal around the base station at the given distance, the signal received by the receiver. Figure 2.8 below shows how the signal level which has been normalized varies with the distance and substracted the empirical model of path loss, which we have talked about in the Path Loss section,

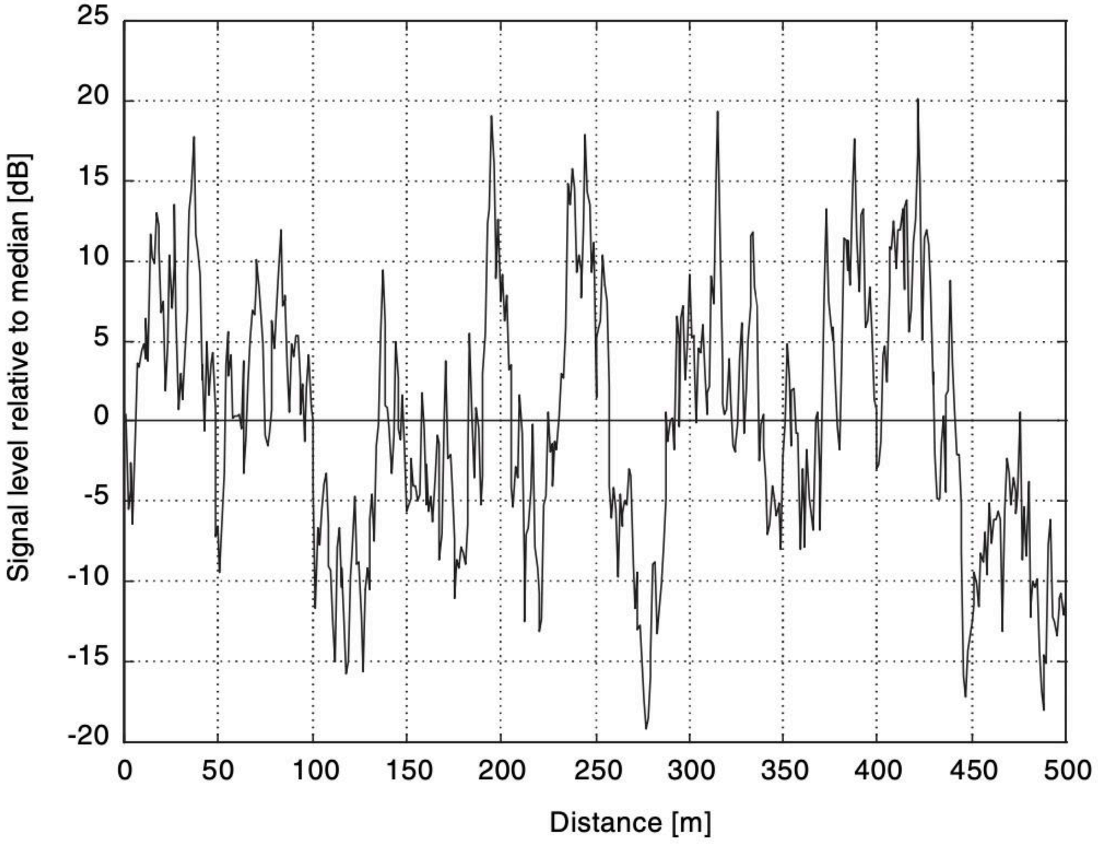


Figure 2.8 Variance of Large-Scale fading with the mobile position at a fixed distance, source[3]

we can see that the Probability density function of shadowing can be represented as log-normal distribution, which is also a Gaussian distribution, the Figure 2.9 below shows the probability density function of the shadowing level measured in decibels.

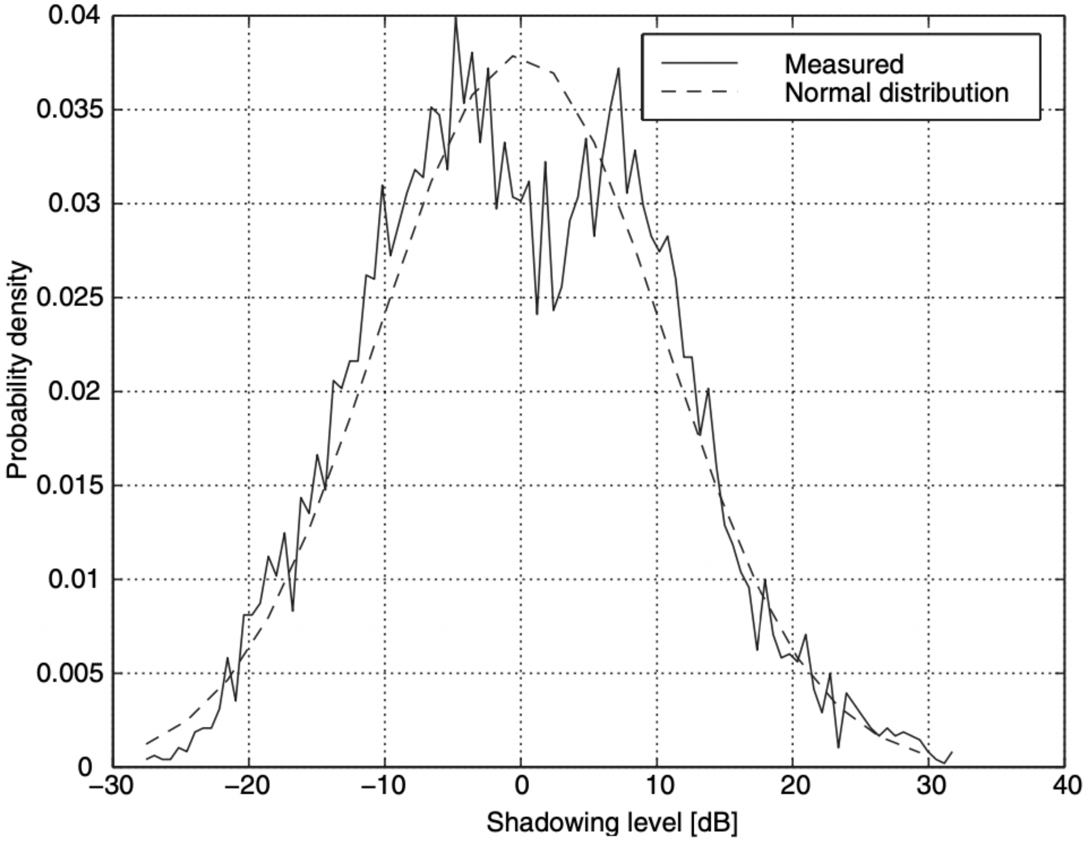


Figure 2.9 Probability Density Function vs. Shadowing level source[3]

The standard deviation of the distribution is called location variability[3] $\sigma_L$, which depends on the area, such as the environment (open area, suburban or urban).

## 2.4 Small-Scale Fading

### 2.4.1 Multipath propagation

Small-Scale fading is caused by multipath propagation, which means the electromagnetic waves radiated from the transmitter can arrive in different paths but the same receiver. These different paths can be caused by the waves reflecting or scattering from objects such as buildings located on the other path where the signal wave is transmitted. Because of these reflections and scattering, serval waves arrive at different times with different amounts of energy loss caused by reflecting and scattering. The waves can have different phases and amplitudes. These will lead to constructive and deconstructive interference on the waves that the receiver receives.

### 2.4.2 NLOS & LOS

The complexity of the multipath propagation means that if we want to predict this type of fading, we need detailed knowledge of parameters in this radio system, such as the characteristics of electromagnetic waves from different paths. So typically, statistical tools are used to predict this type of fading.

There are two different situations of multipath propagation. One is called non-line-of-sight (marked as NLOS), while the other is called line-of-sight (marked as LOS). The difference between NLOS and LOS is if there is a direct path from transmitter to receiver. If so, the situation is categorized as LOS. If not, then the situation is treated as NLOS. Different situations have different statistical tools. The figures below show the LOS (Figure 2.10) and NLOS (Figure 2.11).

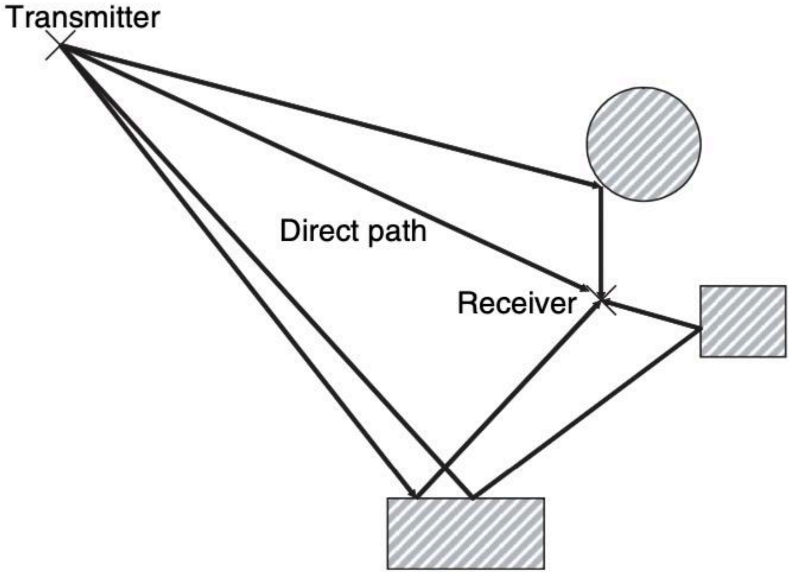


Figure 2.10 Line-on-sight

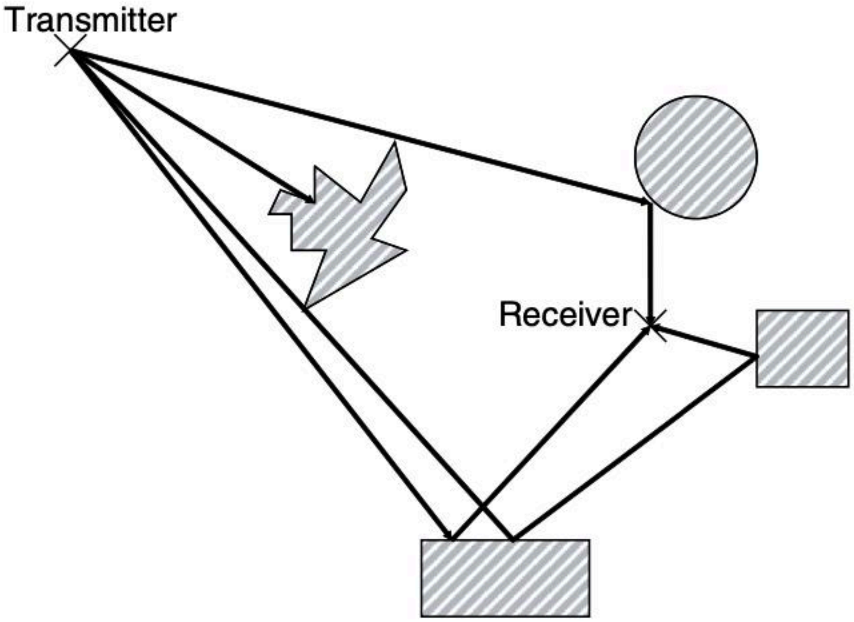


Figure 2.11 None-line-on-sight

### 2.4.3 Rayleigh Distribution

In the NLOS situation, the Rayleigh distribution is a perfect approximation for measuring the fading amplitude of the electromagnetic waves that arrive at the receiver. The Probability Density Function of Rayleigh distribution is described in mathematics below

$$P(r) = \left(\frac{r}{\sigma^2}\right)^{-\frac{r^2}{2\sigma^2}}.$$

The signal of electromagnetic waves can be described in complex number as below[3]

$$\alpha = x + jy$$

where $r$ is the magnitude of $|\alpha| = \sqrt{x^2 + y^2}$ and $\sigma$ is the standard deviation of either the real part of $\alpha$ or the imaginary part.

Since we can consider that there are enough waves in propagation and they are independent from each other, the distribution of either real part or the imaginary part can be considered as normal distribution with zero means and unit standard deviation after normalizing all of the values. The Rayleigh distribution, therefore, describes the magnitude of the waves, which is the fading amplitude distribution.

### 2.4.4 Rice Distribution

In the LOS situation, the receiving signal can be treated as composed of NLOS about random multipath propagation with its Rayleigh Distribution Amplitude and the LOS propagation which takes the main part of the power that arrives receiver, because the propagation power loss under LOS only considers the path loss and shadowing. Therefore it can not be used in the NLOS situation since the distribution is not fitting on either the theoretical inferences we mentioned above or the measurement as shown in Figure below.

Therefore we need to find new statistical tools to predict the impact of the multipath propagation under the LOS situation. The Rice Distribution can describe such fading accurately. The Probability Density Function of Rice Distribution[6] is written below

$$P(r) = \frac{r}{\sigma^2} e^{-(r^2+s^2)/2\sigma^2} I_0\left(\frac{rs}{\sigma^2}\right)$$

Where $\sigma^2$ is same with that of Rayleigh Distribution, the variance of the either real part of the electromagnetic waves or the imaginary part. $s$ is the magnitude of the LOS primary propagation wave. The $I_0$ is the modified Bessel function of the first kind and zeroth order[3]. The Rice probability density function can also be expressed with another parameter, $k$, which is called $Ricefactor$, and the Eq(2.4.4.2) below shows the its definition

$$k = \frac{power\ in\ LOS\ main\ propagation}{power\ in\ random\ NLOS\ propagation} = \frac{s^2}{2 \cdot \sigma^2}$$

The Rice factor describes the ratio of two different parts of the power that arrives receiver. The larger the $k$ value is, the LOS part has more power than the NLOS part which is the multipath propagation. The expression of Rice Distribution with Rice factor is shown below.

$$P(r) = \frac{r}{\sigma^2} e^{-r^2/(2\sigma^2)} e^{-k} I_0\left(\frac{r\sqrt{2}k}{\sigma}\right)$$

Figure 2.12 shows that the probability density function (p.d.f) changes with the k increase.

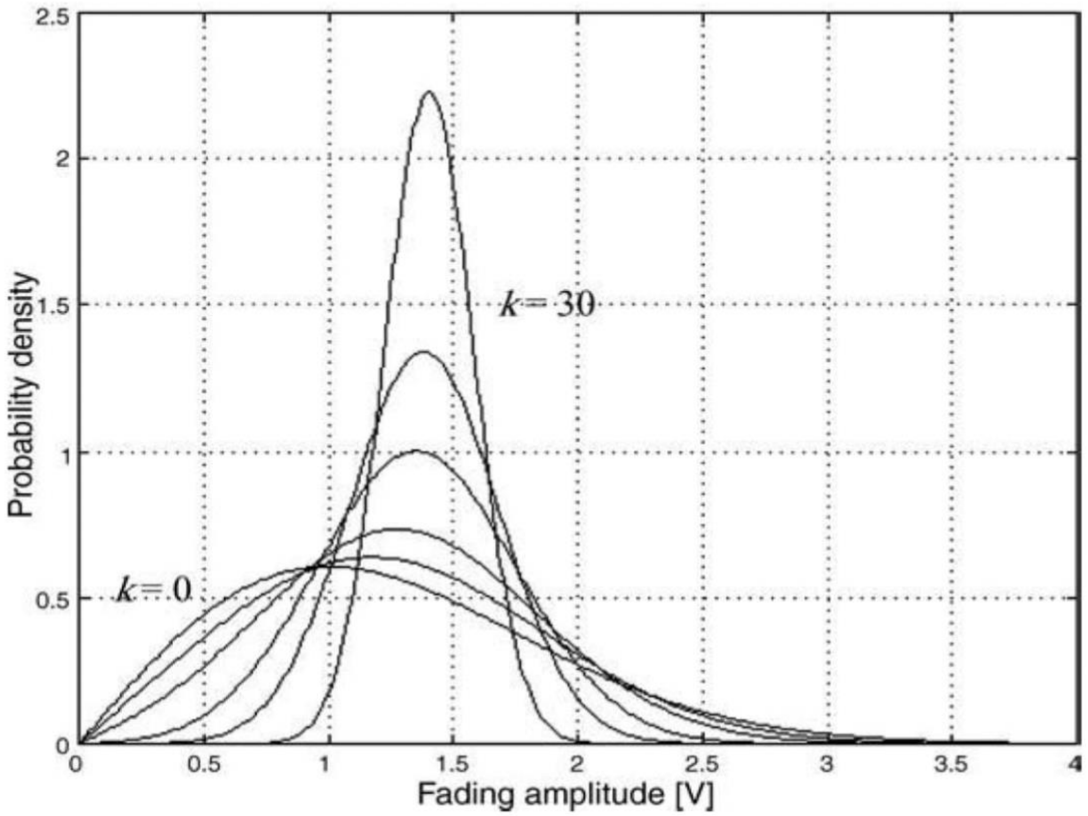


Figure 2.12, Rice p.d.f. for k = 0, 1, 2, 5, 10 and 30, source[3]

From Figure 2.12, we can see that when the k value increases, the mean error rate will decrease.

### 2.4.5 Summary

The multipath propagation causes small-scale fading real world is not two-dimension so that the electromagnetic waves can be radiated in all directions, and scattering can happen. The small-scale fading can have different characteristics with different radiation situations. When it comes to the NLOS(non-line-of-sight), which means there are no direct waves from the transmitter that has radiated to the receiver, we can use Rayleigh Distribution to make the prediction since the pattern of the signal strength follows Rayleigh distribution. However, when it comes to the LOS(line of sight), which means there is a direct wave from the transmitter, then the signal power arrived receiver is the component of the power of the direct wave and the NLOS multipath propagation. Therefore the Rayleigh Distribution is not suitable to describe the situation and Rice Distribution is introduced. Rice distribution not only describes the signal power loss of multipath but also concludes the power ratio from the direct wave and NLOS waves. The higher value k is, which means the power from direct wave propagation is more than that of NLOS waves.

## 2.5 Radio Environment Map

### 2.5.1 Cognitive Radio

Cognitive Radio (written as CR) is one of the wireless communication forms that intelligently detect which channels are used or which are not. So CR can adjust to different communication channels, which is free to avoid interfering with the usage of the channel's primary users (PUs) [7]. CR has the ability to learn from the experience like the usage pattern of PU of the channel and the usage of all channels on the spectrum so that CR can find the appropriate channel to use and improve the quality of service and maximal the usage of each channel.

### 2.5.2 REM

Radio Environment Map (written as REM) is an integrated spatiotemporal database that contains the information of multiple domains, for example, the geographical information, location, radio services regulation and policy, radio frequency source, and the memory of the radio behavior in the past period, the radio devices and sensors that detect the environment and its data. By storing this information in REM, CR can make decisions on this information and does not need to have a highly computation-intensive requirement for the devices from the user side; instead, from the REM server side.

The REM can be split into two parts, Local REM and Global REM. Local REM refers to the network that provides location information and location-based services. In contrast, the Global REM refers to the network that maintains the overview environment of radio in a big areas like cities. The local REM and global REM can share the information in a timely manner so that REM can keep the information update-to-date.

For example, based on our knowledge overview so far, we know that the power received by the receiver can be affected by path loss, large-scale fading, and small-scale fading. REM can store the information about the current environment collected by the sensors and the long-term measured environment data. CR can learn from these data stored in REM and then know about the characteristics of the local environment so that it can choose the appropriate model to predict the coverage of the transmitter with specific signal power and then make the prediction about the quality and the performance of services[7]

### 2.5.3 Summary

Radio Environment Map is the support that is proposed to enhance the capability of cognitive radio. It is a cost-efficient, reliable and flexible way to support the CR. For example, if there is a new regulation or new buildings, simply updating the REM accordingly would save the effort of implementing everything repeatedly because the REM is a multidomain database that contains all the information required for CR. It can also be used to make an appropriate prediction using the data stored in REM for the signal strength. But the books[7][13] have also addressed the issues about the REM those need to be solved and improved as well as the security issues. Such issues as how the local REM server exchanges the data with the global REM server, the privacy issues, security of the user device information as well as the latency of the REM network can affect the performance of the whole system. Also, political resistance is one of the biggest challenges in business, not just technology[7].

## 2.6 Electric Field Integral Equation

Electric Field Integral Equation (written as EFIE) can help the communication system designer to establish the signal coverage of the transmitter. EFIE is not the only way that we can figure out the coverage of the signal, but it is the way that is easy to understand and computer-compactable. After solving the EFIE, we can determine where to put the base station, which can have a broader coverage with less cost.

### 2.6.1 Approximation

Two approximations are used to simplify the whole process.

From the Figure, all electromagnetic waves emitted from the transmitter are reflected by the surface on their way to receivers or observe point, where the surface can be categorized as any type of the area we have discussed in Chapter 2.2. The surface is the terrain profile that describes the detailed geographical information of the area. Also, in the paper, we will consider the 2-D surface only. The first approximation is that the model is a PEC model. The PEC stands for a perfect electrical conductor, meaning there is no current resistance.

The second approximation is the Forward Scattering Approximation. We assume that all the radiation is propagating away from the source transmitter. This is a very important approximation. When the electromagnetic wave from the source transmitter arrives surface, the wave can induce the surface with a current flow. The current can radiate its electrical field to other points on the surface acting like the transmitter. When the induced current radiates to other points, it will also induce the current on other points. The direction of the radiation of any surface point can be any angle given by the terrain profile, but we only consider that the new radiation is forward scattering.

### 2.6.2 Equations

The equation below is the core equation of the whole problem. It describes as follows.

$$E^{Inc}(r) = \frac{\beta\eta}{4}\int_S J\,(r')H_0^{(2)}(\beta|r - r'|)dr'$$

Where $E^{Inc}$(r) is the electrical field strength induced by source (induced segment as source), $J(r')$ is the surface current which is the critical value we would like to know by calculating the Eq(2.6.1.1). Because as long as we know the current of all the points on the surface and we know the source power, we can decide the level of the signal. $r$ is the distance between the source transmitter and the target point on the surface, and the $r'$ is the point induced before the target point. $H_0^{(2)}$ is the Hankel function with the second kind and zeroth order. $\beta$ and $\eta$ is the constant. $dr'$ is the length of the segment of the points on the surface. $S$ is the surface.

Figure 2.12 below shows the details.

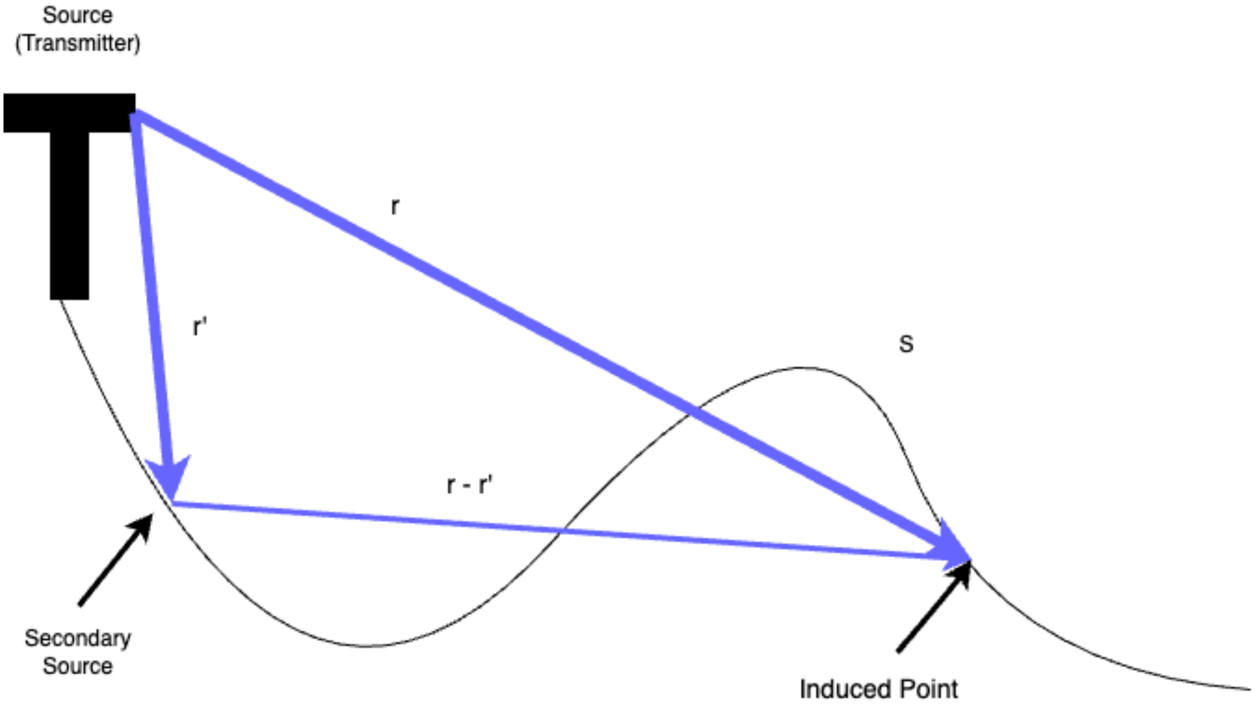


Figure 2.12 Propagation

In Eq(2.6.1.1), $J(r')$ is the only unknown value. The $E^{Inc}(r)$ is defined as

$$E^{Inc}(r) = H_0^{(2)}(\beta|r|)$$

Therefore, after we get the current value ($J(r')$) of each point on the surface, we can then calculate the signal strength of the observation point. The equation below describes the total signal strength or the electrical field strength of the observation point.

$$E^{Total}(r_{obs}) = E^{Inc}(r_{obs}) + \frac{\beta\eta}{4}\int_s J\,(r')H_0^{(2)}(\beta|r' - r_{obs}|)dr'$$

Where $r_{obs}$ is the distance from the source transmitter to the observation point. The equation describes that the total electrical field strength of the observation point is the sum of the electric field induced by the source transmitter and the surface scatter. Figure 2.13 below shows the details

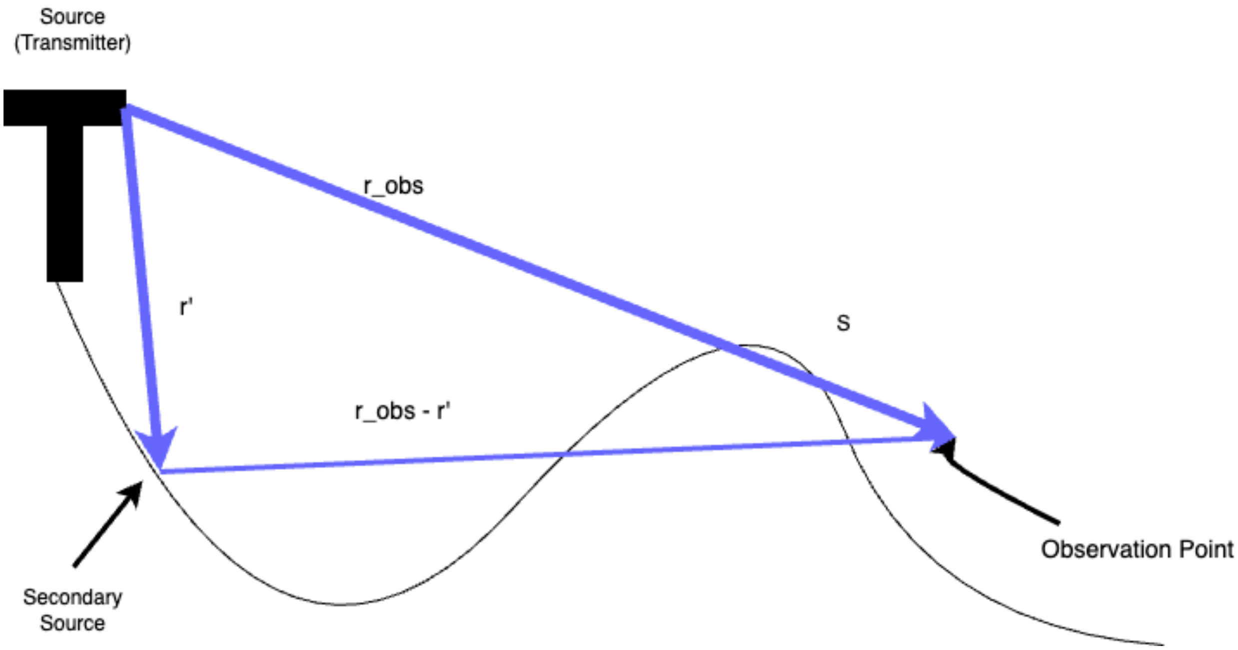


Figure 2.13, Observation point

### 2.6.3 Computer Compatibility

From the mathematics view, it is hard to directly find the explicit formula to solve the $J(r')$. However, we know how to calculate the rest of the components, so we can modify the equation to satisfy how a computer can solve it.

Start from a basic integral equation

$$y(x) = \int_0^x x^2 \, dx$$

When we modify Eq(2.6.3.1) in a computer-compatible way, we can get the following equation

$$y(x) = \sum_{n=0}^{N} x^2 \, \Delta x$$

The Eq(2.6.3.2) is the Eq(2.6.3.1) in discrete from, which is computer compatible. So we can apply the same modification to Eq(2.6.1.1) and get the following equations.

$$E_j = \sum_{i=1}^{i \le N} Z_{ij} J_i$$

Where $Z$ is the $N \times N$ impedance matrix that includes all right right-hand sides2.6.1.1) except the $J(r')$, which is expressed as $J_i$ in Eq(2.7.3.3). $N$ is the number of the segments from the surface. The Eq(2.6.1.1) describes that the induced electrical field strength of point $j$ is the sum of electrical fields scatter from all points to the target point $j$. The Z matrix is summarized and aggregated as following

$$Z_{ij} = \Delta s \cdot \frac{\beta\eta}{4} H_0^{(2)}\left(\beta\left|r_i - r_j\right|\right)$$

Where $i$ represents the source points, $j$ represents the target induced points. Or the self impedance.

$$Z_{ii} = \Delta s \cdot \frac{\beta\eta}{4}\left(1 - j\frac{2}{\pi} ln\left(\frac{1.781\beta\Delta s}{4e}\right)\right)$$

For each $E_j$, we can expend the Eq(2.7.3.3), then we can get the following equations.

$$E_1 = J_1 Z_{11} + J_2 Z_{21} + J_3 Z_{31} + \ldots + J_N Z_{N1}$$

$$E2 = J_1 Z_{12} + J_2 Z_{22} + J_3 Z_{32} + \ldots + J_N Z_{N2}$$

In general, we can get

$$E_N = J_1 Z_{1N} + J_2 Z_{2N} + J_3 Z_{3N} + \ldots + J_N Z_{NN}$$

When we take the approximation we made in the last section, the Forward Scatter Approximation, we can further reduce the expended equations.

$$E_1 = J_1 Z_{11}$$

$$E_2 = J_1 Z_{12} + J_2 Z_{22}$$

Since there will not be the scatter from the points forward to the previous point. So we can calculate $J$ easier by doing the math below.

$$J_1 = \frac{E_1}{Z_{11}}$$

We can get $J_1$ because we have mentioned that $E_1$ and $Z_{11}$ can be calculated directly. As long as we know $J_1$, we can move forward to get $J_2$ as follows.

$$J_2 = \frac{E_2 - J_1 Z_{12}}{Z_{22}}$$

So, in general, we can get the equation below to find all $Jj$ value

$$J_j = \frac{E_j - \sum_{i=1}^{i<j} J_i Z_{ij}}{Z_{jj}}$$

After we get all current values, we can calculate the electric field strength of the observation point above the surface so that we can go further to get the signal strength. From the result, we will have the knowledge of how is the signal strength spread and the coverage of the transmitter.

### 2.6.4 Summary

The EFIE is one way that we can use to solve the problem of finding the coverage of a transmitter. It describes the attenuation we mentioned from section 2.2 to section 2.4. Even though the EFIE is the fundamental way and easy to implement it on a computer, when considering the efficiency of running on a computer, it is a $O(n^2)$ (marked as Big O notation) algorithm which means the time spent on solving will increase quadratically with the length of the input increase. Because of the time consumption of solving EFIE, which means it will take a long time to run for a 2-D terrain profile, EFIE is unsuitable for those big areas or long-distance predictions. It would be suitable for the small or medium areas. It is mentioned in the paper[15], that the EFIE method can have a standard deviation of 3dB at the frequency of 144MHz, and 9dB at the frequency of 1900MHz. The mean errors vary from 5dB to 6dB at a higher frequency range from 970MHz to 1900MHz.

Many papers[8][16] have proposed faster methods to solve the EFIE by using some statistical estimation, which makes a reasonable approximation prediction within a short time.

## 3. Implementation

We are using MatLab as the software for the programming part, and it contains a lot of built-in functions we need during the calculation with the way that other scientists and engineers have optimized. Also, the plot function offered by MatLab is also a part of its advanced applications.

We use the terrain profile which is a 2-D profile that describes the geographical information of the area. The graph below shows the detail of the terrain profile we use.

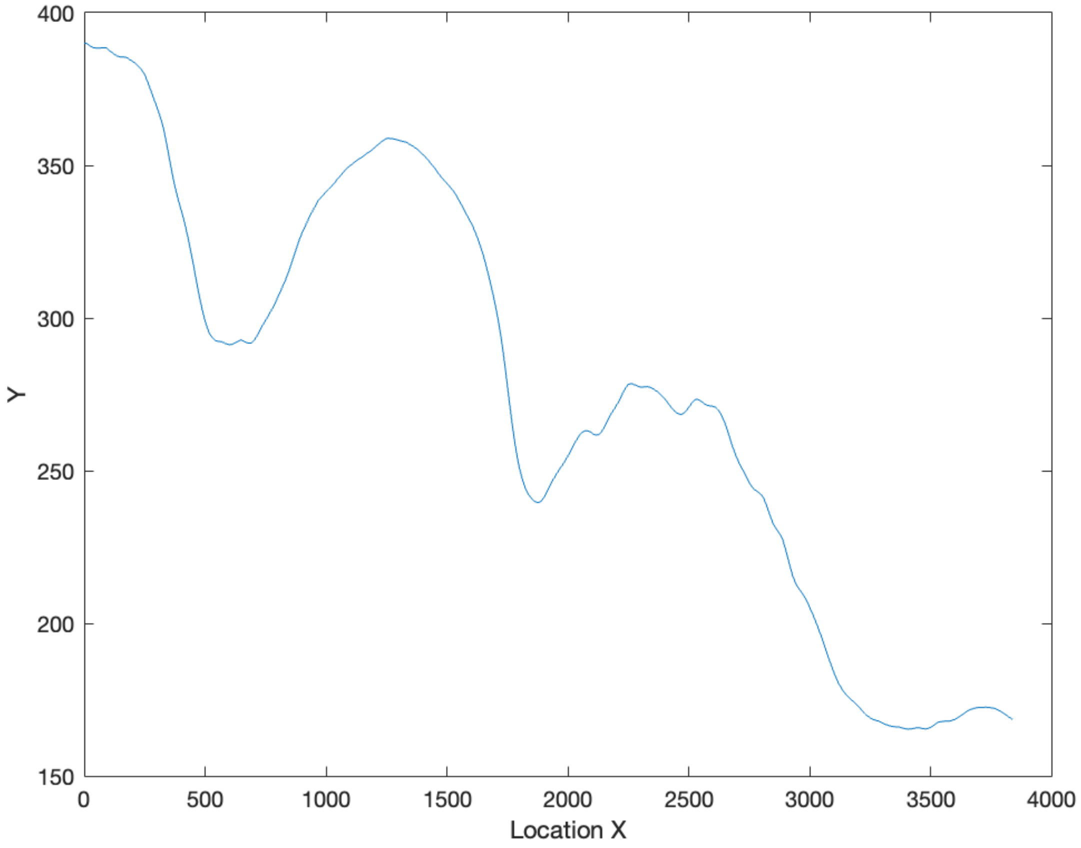


Figure 3.1, Terrain Profile

We can see from the plot that the distance of the terrain profile can be far more than 3500 meters, but in this project, we just use the first 700 meters as our testing terrain profile. If the implementation and result are correct for the first 700 meters, then we can infer that this implementation is suitable for the result of the terrain profile.

## 3.1 Details

This section will discuss the MatLab code for implementing the method. There are some constant values we used in the program. The table below shows the constant we have.

| Constant | Value | Physical Meaning |
|---|---|---|
| u_0 | 12.56637061e-7 | vacuum permeability |
| e_0 | 8.854e-12 | vacuum permeability |
| f | 970 MHz | selected frequency |
| deltas | 0.0733 | quarter of the wavelength, length of segment |

| Constant | Value | Physical Meaning |
|---|---|---|
| transmitter_X | 0 | horizontal coordinate of the transmitter on terrain profile |
| transmitter_Y | 442 | vertical coordinate of transmitter on terrain profile |
| obser_above_surface | 2.4 | observation vertical distance above the surface |
| y | Matrix | vertical coordinate values from the terrain profile |

These constants are some basic physical constants used in propagation

(such as $u_0$ and $e_0$), and the constants we used for this project only(such as $f$ and $deltas$) can vary for different projects and purposes. We can use the constants above to calculate the other values we need to solve the EFIE.

The table below shows some functions used to solve the EFIE in the program and what the functions calculate. All implementation with different approximation and method shares the same code of the functions.

| Function | Description of Functions |
|---|---|
| X | Get the horizontal coordinate segment (which is quarter of wavelength) on the terrain profile |
| Y | Get the vertical coordinate of the segment |
| R_p_q | Distance between two segments p and q |
| R_source_p | The distance between transmitter(source) to another segment on surface |
| R_source_obs | The distance between induced segment to another segment on surface |
| R_source_obs_s | The distance between transmitter(source) to any observation point |
| Z_p_q | impedance value from segment p to q |
| Z_distance | impedance value for a given distance |
| Z_self | Self-impedance value |
| EiRad | Electric field strength scattered from a given distance |

### 3.1.1 EFIE implementation with Forward Scatter

We start to implement the EFIE with the approximation Forward Scatter only. The logic of the whole program is that first, we initialize the current of the first segment, which length is $\frac{\lambda}{4}$, where the $\lambda$ is the wavelength which the following equation can calculate

$$\lambda = \frac{c}{f}$$

Where $c$ the light speed can be calculated with equation

$$c = \frac{1}{\sqrt{u_0 \cdot e_0}}$$

According to Eq (2.6.3.6), we can get the current of the first segment $J_1$.

$$J_1 = \frac{E_1}{Z_{11}}$$

Where $E_1$ is calculated with the function $EiRad$ of which the key parameter is the distance. Then we can move on with implementing the code to execute the Eq(2.6.3.8). A two-layer loop completes the whole process,

$$J_j = \frac{E_j - \sum_{i=1}^{i<j} J_i \, Z_{ij}}{Z_{jj}}$$

Where the inside loop is used to calculate $\sum_{i=1}^{i<j} J_i \, Z_{ij}$ part, the outside loop for calculate $J_i$ values through all of segments we have.

Therefore we can get the current values of each segment. Then we can calculate the overall electric field strength through our data. In total, there are 9059 segments we need to calculate with the first 700 meters of the terrain profile. The electric field strength with Eq(2.6.1.3) which describes the total electric field is the sum of the electric field transmitted from the source transmitter and the electrical field of all segment scattering to the observation points.

Finally, we need to convert the magnitude of the total electric field (the result is in complex form) to decibel.

### 3.1.2 EFIE implementation with Backward Scatter

The steps for solving EFIE with backward scatter are nearly the same as that of EFIE with forward scatter only. EFIE with backward scatters considers both forward scatter, and the backward scatter. This means the segment induced by the power of the electric field from the source can scatter its field to the segments that are on the way away from the transmitter and those back to the source.

Except for the steps we worked on in section 3.1.1, we need to add another two-layer loop to calculate the power scatter backward to the previous segment. Then we can calculate the electric strength and convert its value to decibel.

### 3.1.3 Fast method (FEXM) Implementation

In paper [8] , a fast method based on the EFIE is proposed with the usage of statistical approximation the electric field strength over the group where we need to aggregate 13 points (which is about 1 meter in distance with the selected frequency) together and treat them as one straight line scattering electric field out as a group. And also we need to make another approximation that the electromagnetic waves from the same group arrive at the same time; therefore, there will be no phase shift difference within the group. The rest keeps the same logic as EFIE implementation code. In this fast method, we implement it with the Forward Scatter Approximation.

In the program of this fast method, we still need a two-layer loop to help use solve the EFIE. However, we have grouped segments and estimated that the value of the electric field is nearly constant, so the amount of the computation has been reduced. The logic is the same as with the normal EFIE program. The difference is that when calculating the $\sum_{i=1}^{i<j} J_i Z_{ij}$, we simplify it with the approximation[8].

## 3.2 Iterative Method

Jacobi Method and the Gauss-Seidel method are the two iterative methods to solve linear equations. When we come back to Eq(2.6.3.6) about the EFIE in the form of

$$E_N = J_1 Z_{1N} + J_2 Z_{2N} + J_3 Z_{3N} + \ldots + J_N Z_{NN}$$

We find that the equation of the EFIE in a discrete way is suitable for using the iterative methods to solve.

We will use the equations below to explain how the iterative method work[12]. Assuming that we have a serious linear equation like below

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 = b_3$$

$$a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = b_4$$

We want to find the unknowns $(x_1, x_2, x_3, x_4)$. If we write the equations above in an explicit form as below

$$x1 = [b_1 - (a_{12}x2 + a_{13}x_3 + a_{14}x_4)]\ /a_{11}$$

$$x2 = [b_1 - (a_{21}x1 + a_{23}x_3 + a_{24}x_4)]\ /a_{22}$$

$$x3 = [b_1 - (a_{31}x1 + a_{32}x_2 + a_{34}x_4)]\ /a_{33}$$

$$x4 = [b_1 - (a_{41}x1 + a_{42}x_2 + a_{43}x_3)]\ /a_{44}$$

In summary, the explicit equations can be written as follow.

$$x_i = \frac{1}{a_{ii}}\left[b_i - \sum_{j=1, j\neq i}^{j=n} a_{ij} \cdot x_j\right]$$

The iterative method starts by initializing values for unknown values. In the first iteration, unknown $x_i$ can have an assumed solution with initial values passed to the method. Then we will use the assumed solution for the next iteration to get another estimated solution and keep the same step until the estimated solution has only a tiny difference from the solution from the last iteration.

The method will converge to the actual solution when the matrix of the $a$ is identified as a diagonally dominant matrix, which means the matrix is a square matrix. The magnitude of elements on the diagonal is larger or equal to the sum of magnitude of the rest elements on the row. It can be described by the following equation.

$$|a_{ii}| \geq \sum_{j \neq i} |a_{ij}|$$

for all $i$. However, this condition is sufficient but not necessary for convergence, so it may still work if the matrix is not a diagonally dominant matrix.

We implement two iterative methods, one is the Jacobi method, and the other is the Gauss-Seidel method. Both methods start initializing unknowns with 0s, but the difference is that the Jacobi method will use the estimated solution from the last iteration in the following iteration. In contrast, the Gauss-Seidel method uses the estimated solution within the iteration. For example, when implementing the Gauss-Seidel method, after having finished the estimated value of $x_1$, the new $x_1$ will be used in the equation of estimating $x_2$. Then the new $x_1$ and $x_2$ will be used to estimate the value of $x_3$. Therefore, theoretically, the Gauss-Seidel method will have a faster convergence with less iteration to reach the real solution.

Based on the description and the example we have worked through, we can see many similarities between the equations that explain how the iterative method works and the discrete form of EFIE. Therefore we implement these two iterative methods to solve EIFE where the matrix in Eq(3.2.1) will be the impedance matrix $Z$ in EFIE, and the unknowns are $J_i$ values corresponding to the unknowns $x_i$. $b_i$ can be replaced with $E_i$. Therefore we can calculate the impedance matrix $Z$ first and then calculate the electric field strength matrix $E$ since both works as the function with the key parameter being distance. Finally, we can implement the iterative method to find out $J$ values.

The program implementing Jacobi Method is called $Jacobi$, which takes parameters as below

| Parameter | |
|---|---|
| A | Impedance matrix |
| b | Electric field strength matrix |
| x0 | Initialiezd value, 0 matrix |
| ep | estimate relative error |

the output $y$ is the current.

Gauss-Seidel method has almost the same parameters but matrix $y$ is passed, which is the matrix that records the current value, it is used as the output matrix as well. This is because the Gauss-Seidel method needs to use the new value from the same iteration.

## 4. Result

The figures for running the program are shown below.

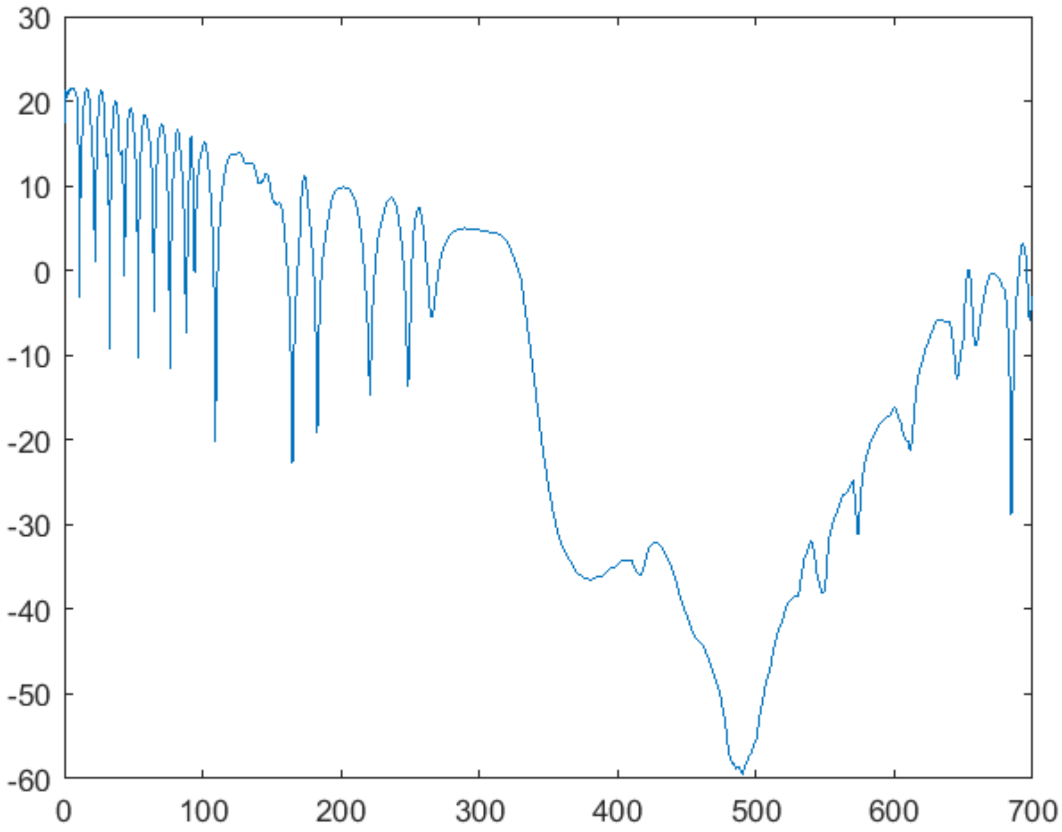


Figure 4.1 Result of EFIE with Forward Scatter only, Electric Field Strength[dB] vs. Distance[m]

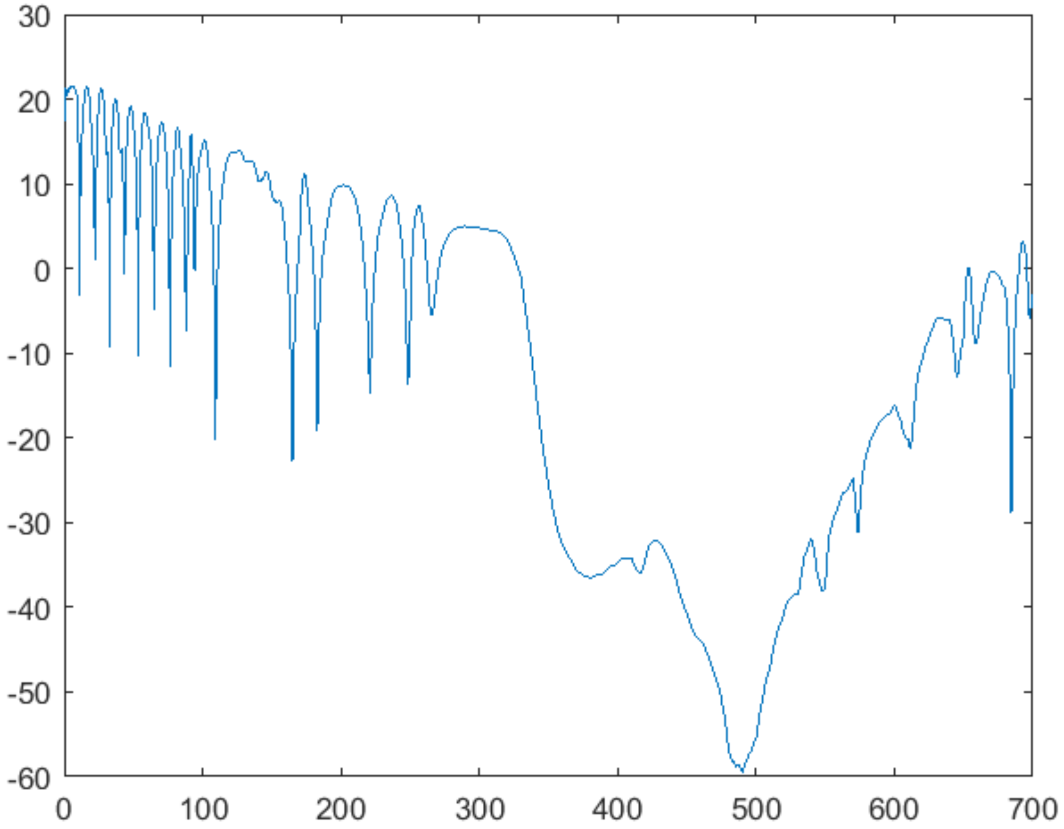


Figure 4.2 Result of EFIE with Both Forward Scatter and Backword Scatter, Electric Field Strength[dB] vs. Distance[m]

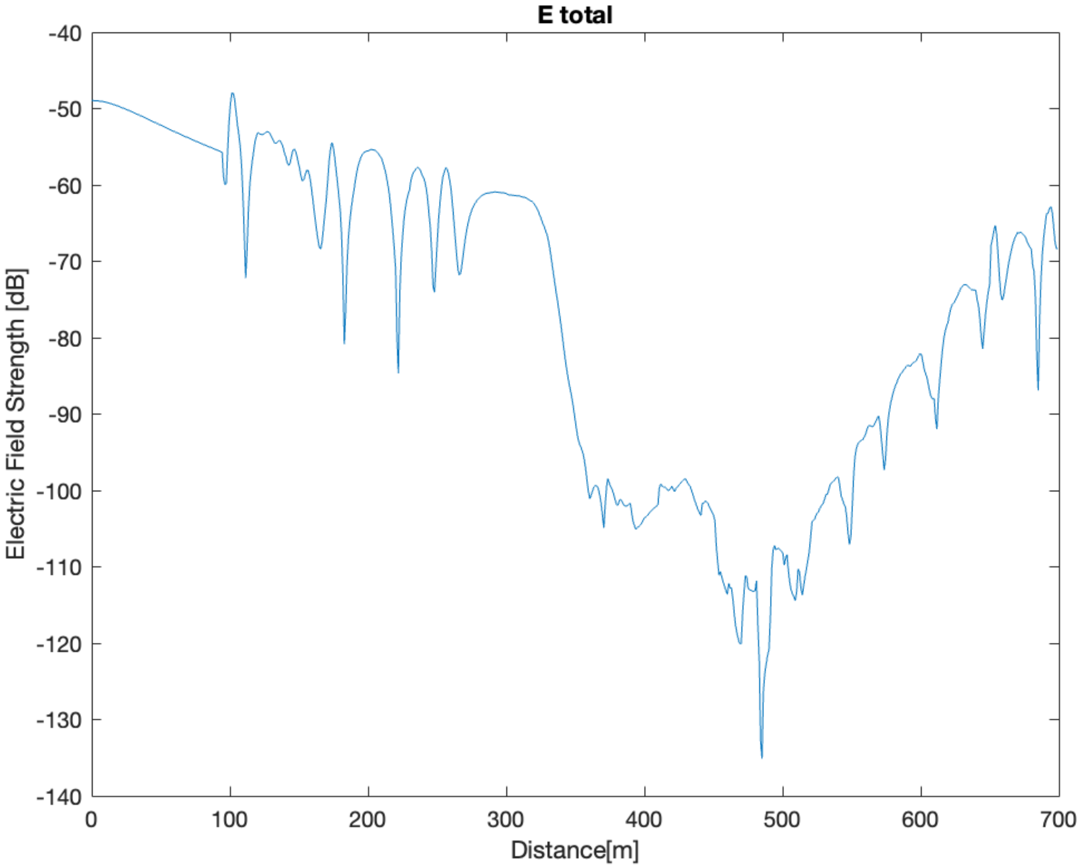
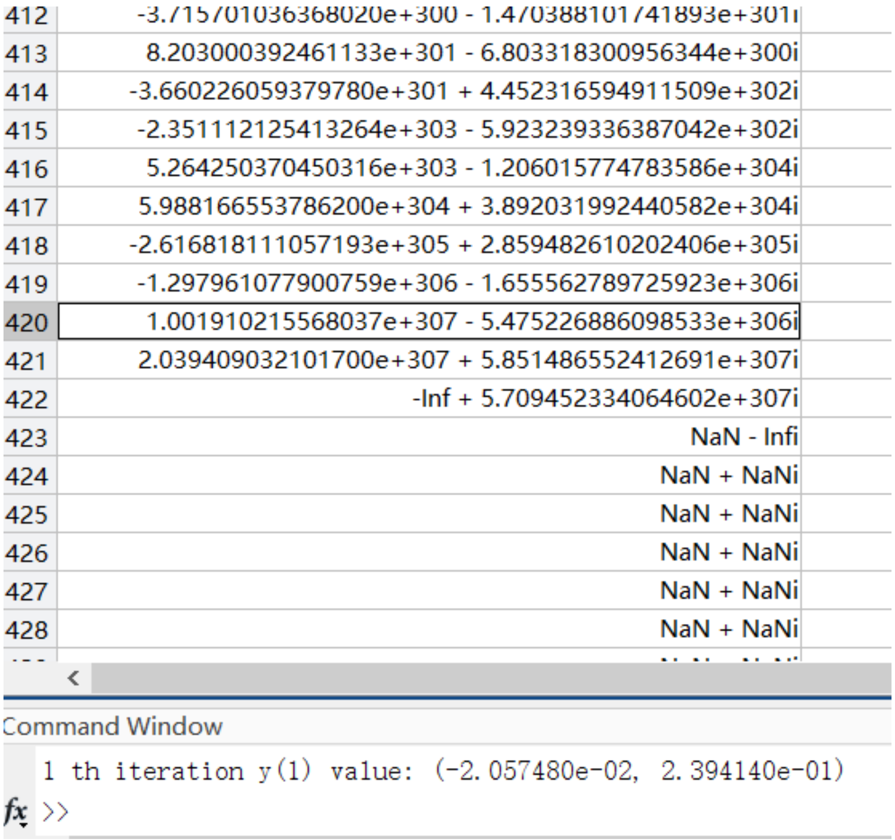


Figure 4.3 Result of fast method, Electric Field Strength[dB] vs Distance[m]

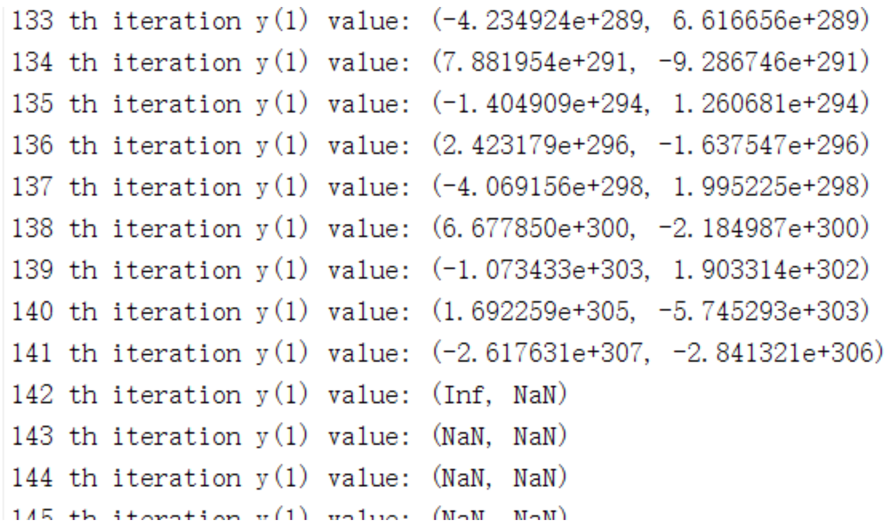


```
412   -3.715701036368020e+300 - 1.470388101741893e+301i
413    8.203000392461133e+301 - 6.803318300956344e+300i
414   -3.660226059379780e+301 + 4.452316594911509e+302i
415   -2.351112125413264e+303 - 5.923239336387042e+302i
416    5.264250370450316e+303 - 1.206015774783586e+304i
417    5.988166553786200e+304 + 3.892031992440582e+304i
418   -2.616818111057193e+305 + 2.859482610202406e+305i
419   -1.297961077900759e+306 - 1.655562789725923e+306i
420    1.001910215568037e+307 - 5.475226886098533e+306i
421    2.039409032101700e+307 + 5.851486552412691e+307i
422                       -Inf + 5.709452334064602e+307i
423                                           NaN - Infi
424                                           NaN + NaNi
425                                           NaN + NaNi
426                                           NaN + NaNi
427                                           NaN + NaNi
428                                           NaN + NaNi
```

Command Window

```
1 th iteration y(1) value: (-2.057480e-02, 2.394140e-01)
>>
```

Figure 4.4 Result of the Gauss-Seidel method



```
133 th iteration y(1) value: (-4.234924e+289, 6.616656e+289)
134 th iteration y(1) value: (7.881954e+291, -9.286746e+291)
135 th iteration y(1) value: (-1.404909e+294, 1.260681e+294)
136 th iteration y(1) value: (2.423179e+296, -1.637547e+296)
137 th iteration y(1) value: (-4.069156e+298, 1.995225e+298)
138 th iteration y(1) value: (6.677850e+300, -2.184987e+300)
139 th iteration y(1) value: (-1.073433e+303, 1.903314e+302)
140 th iteration y(1) value: (1.692259e+305, -5.745293e+303)
141 th iteration y(1) value: (-2.617631e+307, -2.841321e+306)
142 th iteration y(1) value: (Inf, NaN)
143 th iteration y(1) value: (NaN, NaN)
144 th iteration y(1) value: (NaN, NaN)
145 th iteration y(1) value: (NaN, NaN)
```

Figure 4.5 Result of the Jacobi Method

The time of running EFIE of forward scattering only is about 480 seconds using MATLAB 2021b with some methods from external package Mapping Tools on a Windows 10 Laptop with Intel(R) Core i5-8300H @ 2.3GHz. A similar result for running EFIE of both forward and backward scattering is about 490 seconds. As we can see from Figure 4.1 and Figure 4.2, which are the Electric Field Strength versus Distance of EIFE with forward scattering only and with both forward and backward scattering, respectively, there are no differences between both. Compared with the normal way to calculate the EFIE, the fast method has a significantly reduced time to execute the program, which is about 5 seconds. It is a huge improvement. However, even though the tendency of the result calculated by the fast method is similar to that of using the normal method of EFIE, the accuracy is not as good as the latter from Figure 4.3.

The Gauss-Seidel method and the Jacobi method do not work on the EFIE. The estimated solutions can not converge to the real solution. Instead, the estimated solution will go to $Inf$, the keyword in MATLAB that represents the number that is too big to handle. Furthermore, no more valid calculation leads the value of the estimated solution to $NaN$. The Jacobi method reaches the $Inf$ at the 142nd iteration, while the Gauss-Seidel method reaches the $Inf$ during the first iteration.

## 5. Discussion

One reason behind the divergence of the iterative methods for both the Jacobi method and Gauss-Seidel method is that the impedance matrix $Z$ is not a strict diagonally dominant matrix, which means the magnitude of diagonal elements is not greater or equal to the sum of the rest elements on the same row. We have mentioned that the strict diagonally dominant condition is sufficient but not necessary for convergence. It means that when the matrix is not strictly diagonally dominant, we have no idea if it will go convergence or divergence. Based on the implementation of the project, we can see that it will not converge to the real solution.

From the aspect of numerical analysis, the condition number of $Z$ impedance matrix is high, which means even very small changes that happen to the matrix $E$, the value in $J$ will get a huge impact. The condition number measures the ease with which that quantity can be calculated numerically. It is defined as Eq 5.1 below

$$cond(A) = ||A|| \cdot ||A^{-1}||$$

Where A is from $Ax = b$, in our project, A is the $Z$ impedance matrix, $x$ is the J values we want to calculate, and $b$ is the electric field strength matrix. If the condition number is low, we can say the problem is well-conditioned. Otherwise, it will be ill-conditioned.

The condition number of $Z$ impedance matrix is 51.6373 calculated by a build-in function called `cond(A)` in MATLAB. The condition number of the matrix represents that the EFIE is ill-conditioned. This is another reason why the estimated solution for $J$ values diverge to $Inf$.

Therefore, the Jacobi method and the Gauss-Seidel method is not suitable for solving EFIE.

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